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To cite this article: Pedro Carpena and Ana V Coronado 2006 *Eur. J. Phys.* **27** 231

INSTITUTE OF PHYSICS PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **27** (2006) 231–241 doi:10.1088/0143-0807/27/2/006

On the focal point of a lens: beyond the paraxial approximation

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Received 14 September 2005, in final form 6 November 2005
Published 11 January 2006
Online at stacks.iop.org/EJP/27/231

Abstract

In this paper we study a planar-convex lens where the focal point is calculated numerically and analytically beyond the paraxial approximation within the context of geometrical optics. We consider this problem as an appropriate and useful example to fill the gap found in physics and optics courses between the simplicity of the paraxial approximation and the complexity of the theory of aberrations, and it can be used as an introduction to non-paraxial behaviour even when teaching general physics courses. We show in a simple way how beyond the paraxial approximation the focal distance is not unique, and how it depends on the distance of the incoming ray to the optical axis. We show the importance of the caustic surface, which is calculated analytically, and its effect on the position of the point with the highest concentration of light, which is defined as the optimal focal distance of the lens. Finally, we also present some simulations showing light distributions in screens placed at different distances from the lens, to illustrate our results.

1. Introduction

Geometrical optics is the first step in any course on optics because the concept of light rays is usually very intuitive for the students, who can understand the basics of light propagation (reflexion, refraction, etc) without using the more complicated mathematical apparatus of the wave formalism. Within the context of geometrical optics, the behaviour of lenses, mirrors, etc is usually explained in the majority of general physics textbooks [1] and also in simple ray-tracer algorithms [2] considering the paraxial approximation, mainly because of its mathematical simplicity. The study of non-paraxial behaviour (or aberrations) is usually restricted to higher-level and more specific texts [3], for which a good mathematical level is required. Nevertheless, there is a large conceptual gap between both approaches, from the simplicity of the paraxial behaviour to the abstract view of the theory of aberrations.

In this paper, and to fill the above-mentioned gap to some extent, we present the problem of determination of the focal point of a planar-convex lens beyond the paraxial approximation, which is an example of spherical aberration. Our purpose is two-fold: on the one hand the

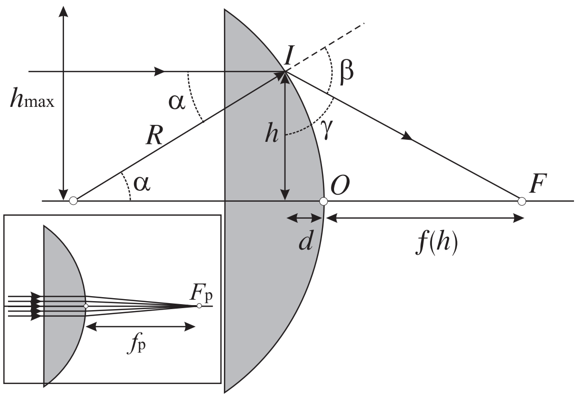

Figure 1. An incoming ray parallel to the optical axis. Inset: the behaviour of the paraxial rays.

mathematical level required in our study is not too restrictive, and therefore the present work can be used as an introductory example of non-paraxial behaviour even in a general physics course. On the other hand, although spherical aberration is discussed qualitatively from a theoretical point of view in many textbooks [3], examples solved quantitatively like the one presented here are not common at all.

In addition, the problem we present in this paper is appropriate for analytical calculation (using approximation methods) but also for a numerical approach, and therefore it can be used as an introductory example of the use of the computer to solve physical problems, especially when simulating light distributions, because the numerical methods required are minimal. This is the reason why we present both the numerical and the analytical approach (see below).

This paper is organized as follows. In section 2, we introduce the planar-convex lens and present the paraxial focal distance. In section 3, we explain how this result is modified for non-paraxial rays, and how the focal distance is not unique. Section 4 is devoted to determination (both numerically and analytically) of the optimal focal point of the lens, by introducing the concept of a caustic surface for which an analytical general expression is given. In section 5, we present the light distributions observed at different distances of the lens obtained by exact numerical calculation.

2. The paraxial approximation

Let us consider a planar-convex lens with an index of refraction given by n, and an incoming light ray parallel to the optical axis, as represented in figure 1. We place the origin of the coordinates at O, the vertex of the lens. The ray enters the lens at a distance h from the optical axis.

At the point of incidence I the ray is refracted, and therefore it must satisfy the law of refraction,

$$n \sin\alpha = \sin\beta, \tag{1}$$

where α and β are the angles of incidence and refraction, respectively (figure 1), and we consider that there is air surrounding the lens ($n_{\text{air}} = 1$). According to the geometry of the problem presented in figure 1, we can easily calculate $\sin\alpha$:

$$\sin\alpha = \frac{h}{R}. \tag{2}$$

In addition, to obtain $\sin\beta$, we have the following angular relation:

$$\frac{\pi}{2} - \alpha + \gamma + \beta = \pi, \tag{3}$$

from where one obtains directly

$$\beta = \frac{\pi}{2} - (\gamma - \alpha). \tag{4}$$

Thus,

$$\sin\beta = \sin\left(\frac{\pi}{2} - (\gamma - \alpha)\right) = \cos(\gamma - \alpha). \tag{5}$$

Using the trigonometric relation $\cos(x - y) = \cos x \cos y + \sin x \sin y$, and taking into account that according to figure 1 we have

$$\sin\gamma = \frac{f + d}{\sqrt{(f + d)^2 + h^2}}, \qquad \cos\gamma = \frac{h}{\sqrt{(f + d)^2 + h^2}}, \tag{6}$$

we obtain finally

$$\sin\beta = \frac{h}{R} \frac{\sqrt{(R^2 - h^2)} + f + d}{\sqrt{(f + d)^2 + h^2}}. \tag{7}$$

If we note that

$$d = R - \sqrt{R^2 - h^2}, \tag{8}$$

we can introduce (2) and (7) into (1) and solve the latter equation for f, which in general will be a function of h, n and R. Up to now the calculation is exact. When considering the paraxial approximation, only the rays very close to the optical axis are taken into account, i.e. rays for which h is very small (or, conversely, lenses for which R is large). In this case it is easy to show that, up to the first order in h, we have:

$$\sin\alpha = \frac{h}{R}, \qquad d \simeq 0, \qquad \sin\beta \simeq \frac{h}{R}\frac{R + f}{f}. \tag{9}$$

Introducing these latter values into equation (1) and solving for f, we obtain the paraxial focal distance f_{p}:

$$f_{\mathrm{p}} = \frac{R}{n - 1}. \tag{10}$$

In this equation there is no dependence of f_{p} on h, and this implies that within the paraxial approximation *all* the incoming rays parallel to the optical axis will converge at the same paraxial focal point F_{p}, placed at a distance f_{p} of the vertex of the lens, as shown in the inset of figure 1. Thus within paraxial behaviour the light forms a perfect point image and aberrations do not exist.

3. Beyond the paraxial approximation

We noted previously that the paraxial approximation is only valid for light rays very close to the optical axis. In this section we study the case of non-paraxial rays, for which we want to obtain the corresponding focal distances. Our system is the same as the one considered in the previous section, and therefore the calculations are identical to the ones we carried out in equation (1) to equation (8), prior to introducing the paraxial approximation in

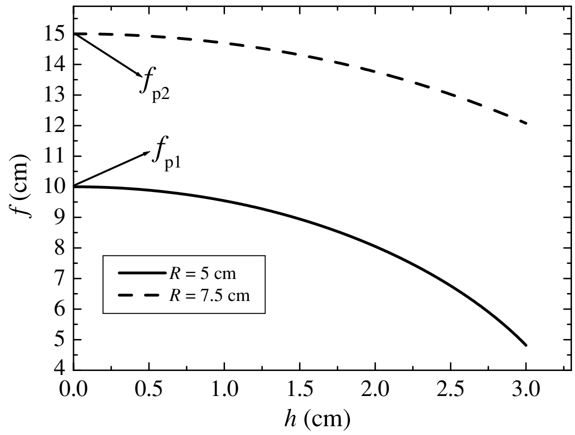

Figure 2. The behaviour of the focal distance f as a function of h for a lens where $n = 1.5$ with two different radii of curvature.

equation (9). In this way, if we introduce (2) and (7) in (1) and solve the latter equation for f, which in general will be a function of h, n and R, we obtain:

$$f(h, n, R) = R\frac{-n\sqrt{R^2 - h^2} + \sqrt{R^2 - n^2h^2} + nR}{n\sqrt{R^2 - h^2} - \sqrt{R^2 - n^2h^2}}. \tag{11}$$

It is straightforward to check that at the limit of the paraxial rays ($h = 0$) we recover the paraxial focal distance f_p:

$$f(0, n, R) = \frac{R}{n - 1} = f_p. \tag{12}$$

Equation (11) indicates that for a fixed lens (i.e. for fixed n and R) there exists a *different* focal distance for any h. Thus beyond the paraxial approximation the focal distance of a lens is not univocally defined, and the corresponding image cannot be a point image. The behaviour of f as a function of h for two values of R and for $n = 1.5$ is shown in figure 3, where the paraxial focal distances are also indicated as a reference. Useful information obtained from figure 2, and not directly evident from equation (11) unless we maximize f as a function of h, is that the maximum focal distance as a function of h is precisely f_p, i.e. the rays very close to the optical axis are the ones which converge furthest from the lens, and any other rays converge closer. From now on for simplicity we omit the dependence of f on n and R, and we write simply $f(h)$.

In principle, equation (11) is valid for any h value provided that the incoming ray is refracted by the lens. It is clear that when total internal reflection is produced at the incidence point I, equation (11) is not valid anymore. The limiting value of h (h_{lim}) separating the region of refraction and the region of total internal reflection can then be calculated as the h value for which the incidence angle α corresponds to the critical angle. If this is the case we have that $\beta = \pi/2$ and equation (1) is now:

$$n \sin\alpha = 1. \tag{13}$$

By using equation (2) one gets

$$h_{\mathrm{lim}} = \frac{R}{n}. \tag{14}$$

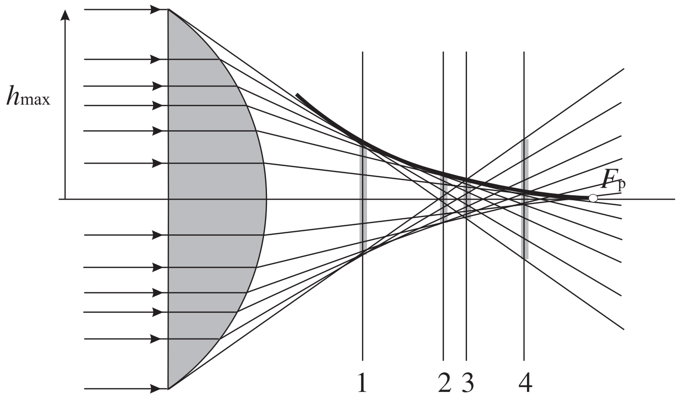

Figure 3. The behaviour of incoming rays parallel to the optical axis. The thick curve represents the caustic surface. The vertical lines represent screens placed at different distances from the lens (see text).

Thus equation (11) is valid for any $h \in [-h_{lim}, h_{lim}]$. Actually, in a real lens, h is limited usually to a much smaller value than h_{lim} in order to make the deviations from the paraxial behaviour small. This limitation is mainly done by controlling the size of the lens h_{max} (see figure 1) in such a way that $h_{max} < h_{lim}$, or even $h_{max} \ll h_{lim}$. In the following, we consider that the incoming rays propagate with $h \in [-h_{max}, h_{max}]$, and that in this range equation (11) is always correct.

4. Optimal focal distance of the lens and the caustic surface

We have discussed already that equation (11) implies that there is no single focal distance beyond the paraxial approximation, and that for an incoming ray parallel to the optical axis, the larger the h, the shorter the corresponding focal distance $f(h)$. Thus, the minimum focal distance is given by $f(h_{max})$, and the maximum focal distance is precisely the paraxial focal distance f_p. The behaviour of several rays parallel to the optical axis in this case is shown qualitatively in figure 3.

In this context and as we mentioned above, it is clear that a point object placed in the optical axis very far from the lens will not produce an image point placed at the focal point F_p. Instead, if we put a screen after the lens perpendicular to the optical axis, we would observe a spot of light of finite (not zero) size. The size of this spot depends on where the screen is placed. In figure 3 we show, as vertical lines, four hypothetical screens placed at different distances from the lens. With a grey rectangle we indicate in each case the size of the corresponding spot of light we would observe in these screens, obtained simply as the area of the screen reached by the rays.

We could define operationally the *optimal focal point of the lens*[1] as the point where one has to put the screen in such a way that the observed spot of light presents the minimum size, i.e. with the highest average concentration of light. We can discuss qualitatively where this point is. Let us consider a screen as in case (1) in figure 3, which is placed close to the lens. It is clear that the spot of light is large simply because the rays have not converged yet. If we move the screen to the right (as in case (2) in figure 3), some of the rays have converged already and the size of the spot decreases. If we displace the screen a little bit more to the right

[1] The image blur in this point is usually called the *circle of least confusion*. See, for example, reference [3], page 525.

(case (3) in figure 3) the size of the spot decreases even more. However, case (3) precisely corresponds to the critical position of the screen where the smallest spot of light is obtained, and thus to the optimal focal distance, because if we continue moving the screen to the right, the rays coming from $\pm h_{\max}$ are now farthest from the optical axis, producing a larger spot of light, as in case (4) in figure 3. How can this optimal focal distance then be calculated quantitatively? For any position of the screen to the left of position (3), the size of the spot of light is given by the *caustic surface*, i.e. the envelope of all the rays emerging from the lens, which is shown as a solid thick line in figure 3. For any position of the screen to the right of position (3), the size of the spot of light is given by the rays coming from $\pm h_{\max}$. Thus, the critical position (3) (and thus the optimal focal distance) is given by the point where the caustic surface intersects the ray coming from $-h_{\max}$. It is obvious that the problem is symmetric, and that the optimal focal point is also given by the intersection between the lower branch of the caustic surface and the ray coming from $h_{\max}$.

The determination of this optimal focal point can be done numerically in a simple way, but also analytically. In order to do both types of calculation, we need first to know in general the equation of a generic incoming ray travelling at distance h from the optical axis. Let us consider the vertical axis as the y axis, and the horizontal axis as the x axis, and let us put the origin of coordinates at the vertex of the lens, as in figure 1. For a generic ray we already know two points of its trajectory. The first one is the incidence point I, and according to the geometry of the problem (see figure 1) its coordinates are $(\sqrt{R^2 - h^2} - R, h)$. The second one is the point where this ray intersects the optical axis (now the x axis), because its coordinates are $(f(h), 0)$, where $f(h)$ is the focal distance given by equation (11). Using these two points it is straightforward to obtain the equation of the trajectory of the ray $(y(x, h))$ which of course is a straight line:

$$y(x,h) = \frac{Rh(\sqrt{(R^2 - n^2h^2)} + nR - n\sqrt{(R^2 - h^2)})}{\sqrt{(R^2 - n^2h^2)}\sqrt{(R^2 - h^2)} + nh^2} - \frac{h(n\sqrt{(R^2 - h^2)} - \sqrt{(R^2 - n^2h^2)})}{\sqrt{(R^2 - n^2h^2)}\sqrt{(R^2 - h^2)} + nh^2} x. \tag{15}$$

This equation gives the distance y from the optical axis of a generic ray (i.e. with a given h value) at any position x of its trajectory once it has been refracted by the lens. From this result, it is possible to find the optimal focal distance both numerically and analytically, as we illustrate in the following.

4.1. Numerical solution

The numerical solution of the problem is then easy from now on. Let us consider a dense set of N rays (with very large N) with h distributed in the interval $[0, h_{\max}]$. For a given h (i.e. for a particular ray) the corresponding trajectory is given by (15). Wherever the optimal focal point is, it must be in the interval $[f(h_{\max}), f_p]$ because these are, respectively, the smallest and largest focal distances of all the incoming rays. Thus, we select again a dense set of x values with $x \in [f(h_{\max}), f_p]$. For any x, we study $y(x, h)$ as a function of h using (15) and we obtain the maximum y value of all the rays at point x. This maximum value is precisely the value of the caustic surface at point x (see figure 3). If we proceed in this way in the whole interval $[f(h_{\max}), f_p]$, we obtain numerically the caustic surface. The optimal focal point can then be found numerically as the intersection point of the caustic surface and the ray coming from $-h_{\max}$.

A numerical example of the results of this algorithm is shown in figure 5, where we consider a lens where $R = 10$ cm, $n = 1.5$ and $h_{\max} = 3$ cm. In this case, $f(h_{\max}) = f(3) \simeq 17.89$ cm and $f_p = 20$ cm, so we consider x values in the range $[17.89, 20]$ cm. We show

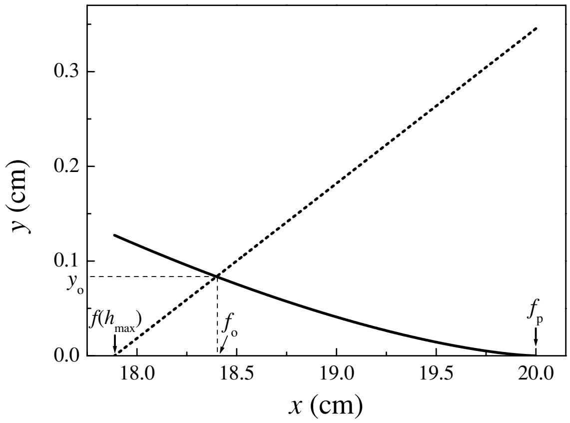

Figure 4. Numerical example to determine the optimal focal distance f_o of a planar-convex lens where $R = 10$ cm, $n = 1.5$ and $h_{max} = 3$ cm. The solid thick curve represents the caustic surface obtained numerically. The dotted line represents the ray coming from $-h_{max} = -3$ cm. The intersection point gives the optimal focal distance f_o and the minimum radius y_o of the spot of light.

with a solid thick line the caustic surface (actually, the upper branch) and with a dotted line the trajectory of the ray coming from $-h_{max} = -3$ cm, obtained directly from (15). For this example, we find numerically that the caustic surface and the ray coming from $-h_{max}$ intersect at $x = 18.398$ cm, and therefore here precisely is the optimal focal point F_o or, if we prefer, the optimal focal distance (f_o) is equal to $f_o = 18.398$ cm. From the same plot we can obtain the size of the spot of light we would observe in a screen as a function of the position x of the screen. From F_o to the left, the size (the radius) of the spot is given by the caustic surface, while from F_o to the right, the size is given by the ray coming from $-h_{max}$. In the example shown in figure 5, the radius of the spot in $F_o(y_o)$ is $y_o = 0.084$ cm, which is of course the minimum possible value. As a reference, the size of the spot in $f(h_{max})$ is 0.127 cm, while in the paraxial focal point it is 0.345 cm.

4.2. Analytical solution

In the previous section, we have explained how to numerically solve the problem, and we have solved an example. In this section we try to solve the problem analytically, and once this is done we will compare both results. It is clear that the most difficult part in the analytical derivation is the obtention of the caustic surface, so we start by doing this calculation.

To obtain the caustic surface, we restrict ourselves to its upper branch, i.e. we calculate the envelope of all the rays with positive h, and thus with $h \in [0, h_{max}]$ (see figure 4). At any x point the caustic surface is given by the maximum value of the y coordinates of all the rays with $h \in [0, h_{max}]$. Thus we need to maximize y as a function of h to obtain which ray (which h value) gives the caustic surface at point x. The distance y of a ray from the optical axis at point x for a generic ray (for any h) is given in equation (15). In general, we see that y is a function of x, h, n and R, but as we consider that our lens is fixed, we assume that R and n are constant, and we write $y(x, h)$. To maximize y as a function of h, we impose the condition

$$\frac{\partial y(x, h)}{\partial h} = 0. \tag{16}$$

This partial derivative can be calculated because $y(x, h)$ is given in (15), so in principle equation (16) could be solved to find the h value which maximizes y for any x. The problem is that the dependence of y on h in (15) is not simple, and therefore the dependence of its derivative is also complicated. To simplify the problem, we take the derivative (actually, only the numerator is needed, because of (16)) and we expand it in a Taylor series in powers of h up to the order h^2. If we do so, and we consider that this expansion (the numerator) must equal 0 according to (16), we arrive at

$$(R^3(xn - x - R)) + \left(\tfrac{1}{2}Rn(-x - R + 3Rn + xn)\right)h^2 = 0, \tag{17}$$

for which the corresponding positive solution (we are restricted to the interval $[0, h_{\max}]$) is

$$h_C(x) = \sqrt{2}R\sqrt{-\frac{(xn - x - R)}{n(-x - R + 3Rn + xn)}}. \tag{18}$$

Note that we call h_C to the solution of (17) because it is the h value which maximizes y for any x, and thus gives the value of the caustic surface $C(x)$, which can then be obtained as

$$C(x) = y(x, h_C(x)), \tag{19}$$

where $y(x, h)$ is the one given in (15). If we introduce (18) into (15) according to (19), we obtain a large expression for $C(x)$. Again, we can simplify the expression by expanding $C(x)$, although now it is more convenient to expand $C(x)$ in powers of $R/(n - 1) - x$. If we do so, and we consider only the first two terms in the expansion and simplify, we get

$$C(x) = \frac{2\sqrt{6}}{9n\sqrt{R}}(R - (n - 1)x)^{3/2} + \frac{\sqrt{6}(3n^2 + 5n - 5)}{54(n\sqrt{R})^3}(R - (n - 1)x)^{5/2}, \tag{20}$$

which is our final result for the caustic surface $C(x)$. Note that to obtain $C(x)$ we have performed an expansion in powers of $R/(n - 1) - x$, but as according to equation (10) $R/(n - 1)$ is precisely the paraxial focal distance f_p, our expression for $C(x)$ will be very accurate in the vicinity of the paraxial focal point F_p, which is precisely the region of interest (see figure 4). Actually, even the first term of the right-hand side of (20) is very accurate to describe the caustic surface, the second term being a correction that should be taken into account far enough from F_p. To show the validity of equation (20), in figure 5 we show the exact caustic surfaces obtained numerically for three different lenses as well as the corresponding caustic surfaces given analytically by (20).

Once we have the expression for the caustic surface $C(x)$ given in (20), determination of the optimal focal distance is simple, because the optimal focal point is given by the intersection between the caustic surface $C(x)$ and the trajectory of the ray coming from $-h_{\max}$, $y(x, -h_{\max})$, obtained from (15). Thus, one has to solve the following equation:

$$y(x, -h_{\max}) = C(x). \tag{21}$$

The analytical solution of this equation is not simple. Even in the case of considering the simplified version of $C(x)$, i.e. the first term of the right-hand side of (20), one has a cubic equation in x with complicated coefficients. It is better to solve (21) numerically. If we consider the same example as the one considered before, ($R = 10$ cm, $n = 1.5$ and $h_{\max} = 3$ cm), the solution of (21) is $x = f_o = 18.402$ cm, which is practically identical to the value obtained in the previous section solving the whole problem numerically ($x = f_o = 18.398$ cm). Thus, the graphical solution of equation (21) is identical to the one shown in figure 5.

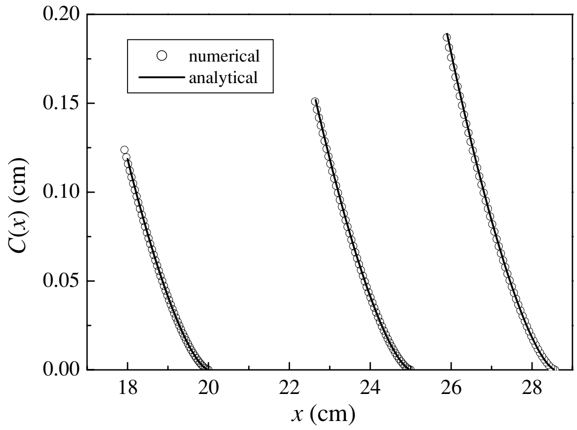

Figure 5. The caustic surfaces obtained numerically (solid curves) and analytically (circles) for three different lenses. The numerical values of the parameters are, from left to right, $R = 10$, 15 and 20 cm, and $n = 1.5$, 1.6 and 1.7 respectively.

5. Light distribution

In a previous section we have addressed the problem of finding the point where the spot of light presents the minimum size, which by definition is the optimal focal point. Now we ask in general about how the light is distributed in the screen, i.e. how the rays that reach the screen are distributed, depending on the distance of the screen to the lens. This problem can be simulated numerically in an exact form using equations given in previous sections, and is very illustrative. It is enough to consider a large number N of incoming rays parallel to the optical axis uniformly distributed[2] in the lens. Due to the symmetry of rotation of the problem with respect to the optical axis, a generic incoming ray reaches the circular lens at a distance ρ from the optical axis and with a polar angle ϕ, such that $\rho \in [0, h_{\max}]$ and $\phi \in [0, 2\pi]$. Given a screen placed at a distance x from the lens, this generic ray will impact the screen at polar coordinates $(y(x, \rho), \phi)$, where $y(x, \rho)$ is given exactly in (15). In figure 6 we show the results of this simulation where $N = 10\,000$ rays for a planar convex lens where $R = 10$ cm, $n = 1.5$ and $h_{\max} = 3$ cm. In this figure, we plot the light distribution in a screen placed at six different distances from the lens, taking into account that each white spot corresponds to a ray. The horizontal and vertical scales are the same for the six cases shown in figure 6, each one corresponding to a square of 0.35 cm × 0.35 cm, and the corresponding distances are: (a) $x = 16$ cm, (b) $x = 17.5$ cm, (c) $x = 17.9$ cm, (d) $x = f_{\rm o} = 18.398$ cm, (e) $x = 19$ cm and (f) $x = f_{\rm p} = 20$ cm.

Cases (a) and (b) in figure 6 correspond qualitatively to case (1) in figure 3. None of the rays have converged yet, and the bright circle limiting the spot in both cases corresponds to the caustic surface, the diference being that in case (b) the rays are closer to convergence. Case (c) corresponds qualitatively to case (2) in figure 3. The bright external border is again

[2] To generate rays, or in general, points uniformly distributed in a circle, it is convenient to use polar coordinates (ρ, ϕ). Nevertheless, one has to be careful in doing this. At first, if the radius of the circle is R, one may think that it is enough to generate random points (ρ, ϕ) with ρ uniformly distributed in $[0, R]$ and ϕ uniformly distributed in $[0, 2\pi]$. Although this assumption is correct for ϕ, it is wrong for R, because this procedure leads to a higher concentration of points close to the origin (i.e. a non-constant surface density) due to the fact that the elemental surface increases with ρ. To avoid this spurious effect, one has to generate ρ values using a probability distribution of the type $P(\rho) = 2\rho/R^2$.

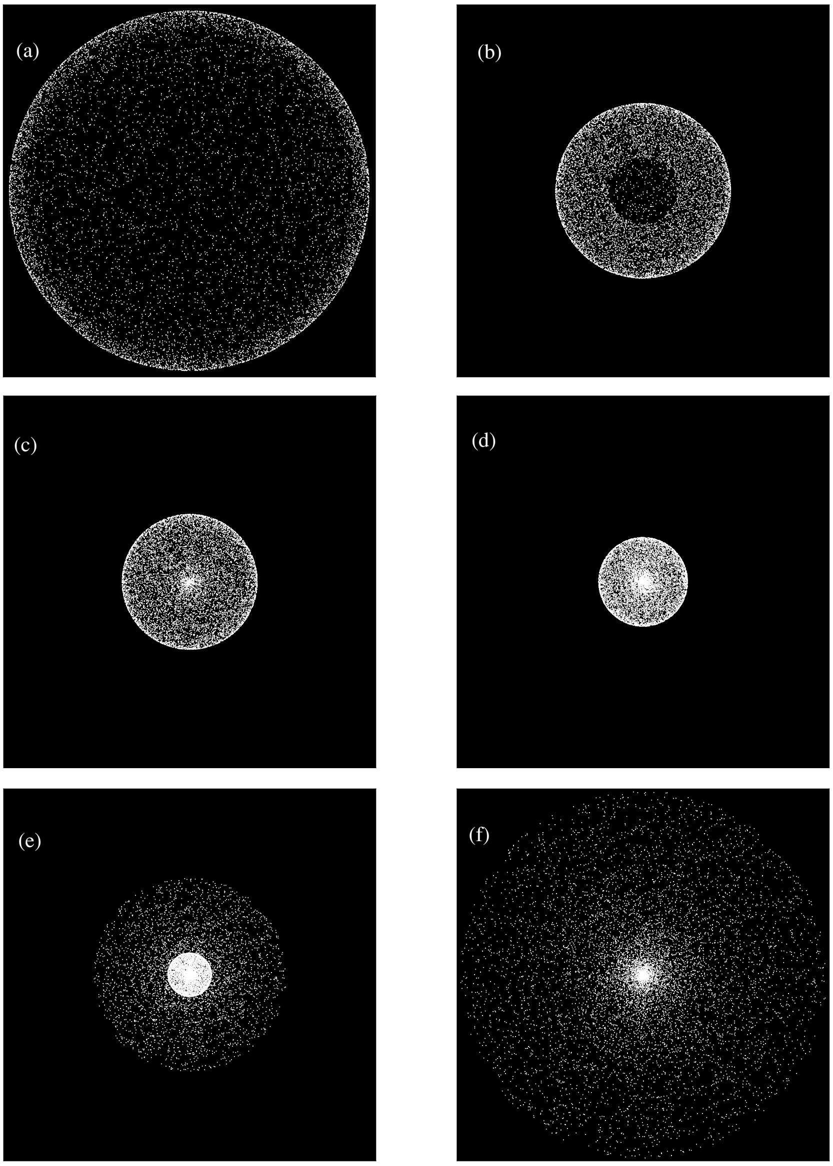

Figure 6. Distributions of light in screens placed at different distances from the lens (see text). The optimal focal distance corresponds to case (d). The paraxial focal distance corresponds to case (f).

the caustic surface, and as some of the rays have converged already, a bright spot at the centre (the optical axis) can be seen. Case (d) corresponds to the optimal focal distance (case 3) in figure 3. Now the size of the light distribution is reduced to its minimum size, the bright external circle again represents the caustic surface, and the bright spot at the centre is produced because many rays are converging in that region. Case (e) corresponds to an intermediate position between the optimal focal point and the paraxial focal point, qualitatively equivalent to case (4) in figure 3. Now the caustic surface is closer to the optical axis, and the bright circle is then smaller than in previous cases. But there are rays which converged before and which now are far from the optical axis forming the cloud of points surrounding the bright circle. Case (e) corresponds to the paraxial focal distance. Now, the caustic has collapsed into the optical axis, and therefore the bright spot at the centre is produced by the paraxial rays. The cloud of points around the bright centre is larger than before, because the rays which converged first are now very far from the optical axis.

Although the results shown in figure 6 are obtained for a particular example, the shapes of the different spots are rather general. Using a simple magnifying glass, for example, students can experiment with the sun and a piece of paper used as a screen by observing the spots of light formed when varying the relative distance between the paper and the magnifying glass, and can compare the observations with the spots shown in figure 6. Of course when doing this the students have to be warned about the adequate precautions required to use magnifying glasses to focus sunlight (i.e. to prevent the direct focusing of sunlight into the eyes, or to prevent the risks of burning objects). In addition, and as the computer simulations needed to produce figure 6 are rather simple, it is a good opportunity for students to use the computer and compare the results with real observations.

Determination of the optimal focal point of a lens is of fundamental importance in many optical systems, such as in astronomic devices. When taking pictures of far-away stars the film must be placed precisely at the optimal focal point of the telescope in order to observe the smallest spot of light (as shown in case (d) of figure 6), and therefore the maximum resolution[3].

6. Conclusions

We have presented an example of an optical system studied within the context of geometrical optics, but going beyond the paraxial approximation. The problem we have solved corresponds to determination of the optimal focal distance of a planar-convex lens. This example can be useful as an introduction to the theory of aberrations, because the mathematical level required is not restrictive. We have shown that, beyond the paraxial approximation, the focal distance is not unique, and also the importance of the caustic surface in order to determine the optimal focal distance of the system, and its relevance in many optical systems. We have presented the analytical and the numerical solution of the problem, and also computer simulations of the light distributions at different distances from the lens. This example is also appropriate to illustrate the use of the computer to simulate physical problems, because the numerical concepts involved are very simple, and also because the students can compare the results of the simulation with real observations carried out with a simple magnifying glass.

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[3] This resolution corresponds to the geometrical optics behaviour. Of course the resolution is also limited by the difraction effects when the wave nature of light is considered.