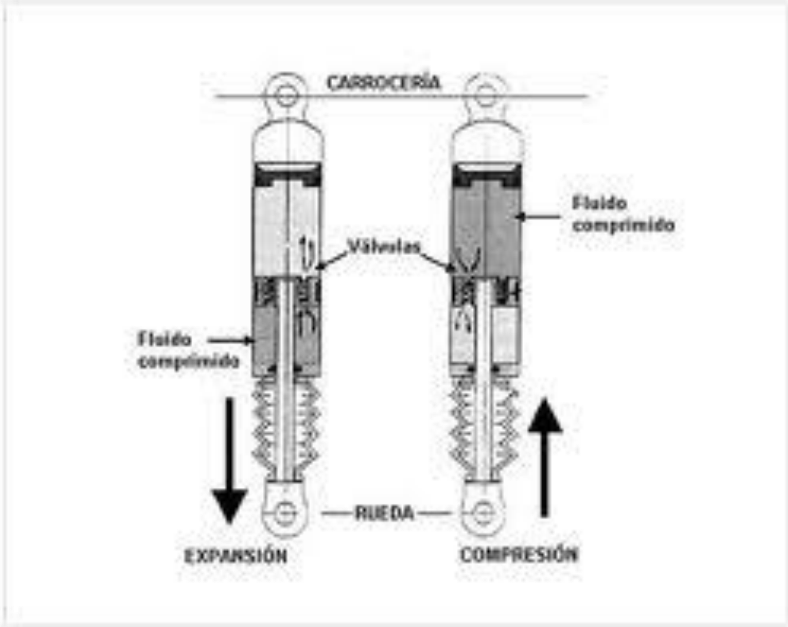
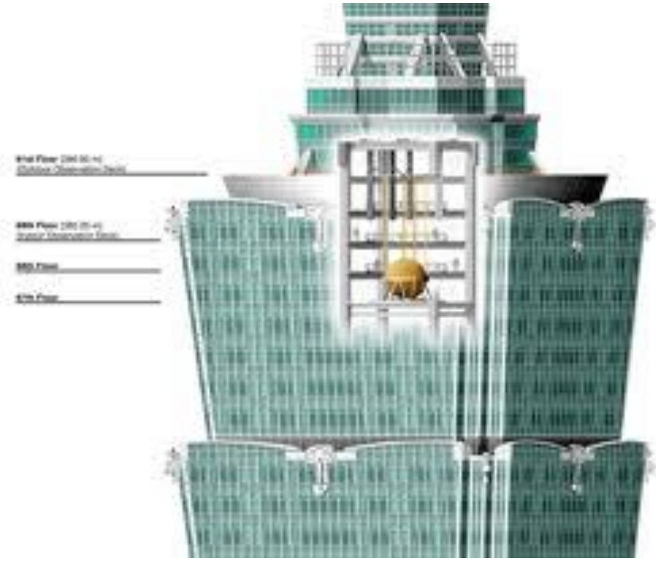
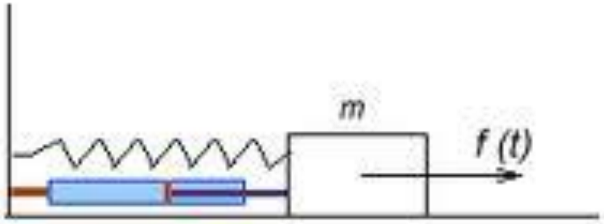
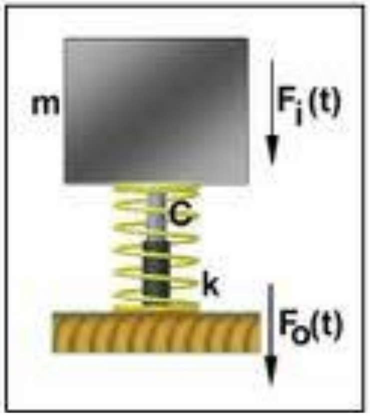
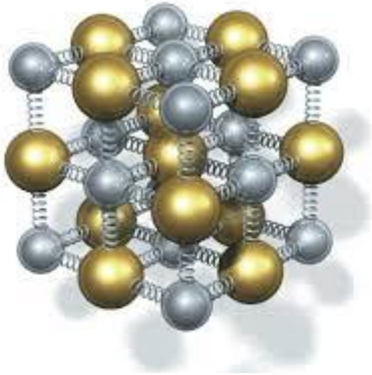
# Oscilaciones Forzadas





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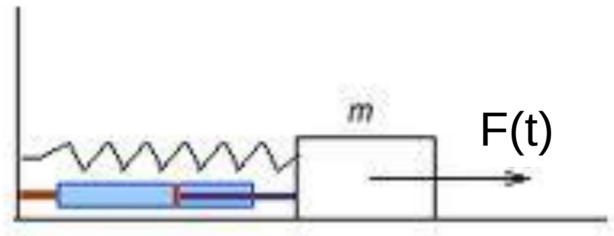


$$\vec{F}_e + \vec{N} + \vec{F}_v + m\vec{g} + \vec{F}(t) = m\,\vec{a}$$

*donde*

$$\vec{F}(t) = F_0 \sin(\omega t)\,\hat{i}$$

La ecuación que el sistema satisface es:

$$\ddot{x} + k_1 \dot{x} + \omega_0^2 x = f_0 \sin(\omega t)$$

donde

$$k_1 = \frac{k_v}{m}$$

$$\omega_0^2 = \frac{k_e}{m}$$

$$f_0 = \frac{F_0}{m}$$

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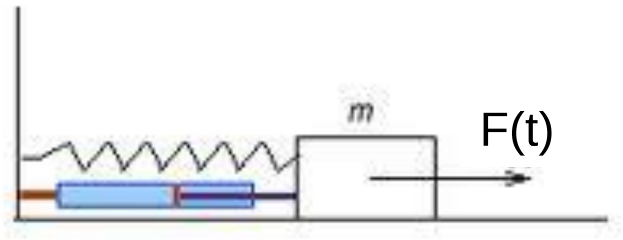


La ecuación que el sistema satisface es:

$$\ddot{x} + k_1 \dot{x} + \omega_0^2 x = f_0 \sin(\omega t)$$

Proponemos como solución:

$$x = x_H(t) + x_P(t)$$

Donde:

$$x_H = Ce^{-\alpha t} \sin(\Omega t + \delta) \quad donde$$

$$\alpha = \frac{k_v}{2m}$$

$$\Omega = \sqrt{\omega_0^2 - \alpha_0^2}$$

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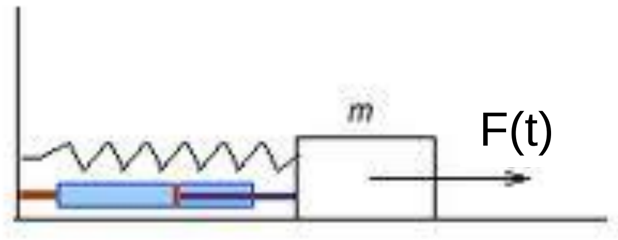


La ecuación que el sistema satisface es:

$$\ddot{x} + k_1 \dot{x} + \omega_0^2 x = f_0 \sin(\omega t)$$

Proponemos como solución:

$$x = x_H(t) + x_P(t)$$

y para el segundo término tenemos

$$x_P = B_1 \sin(\omega t) + B_2 \cos(\omega t)$$

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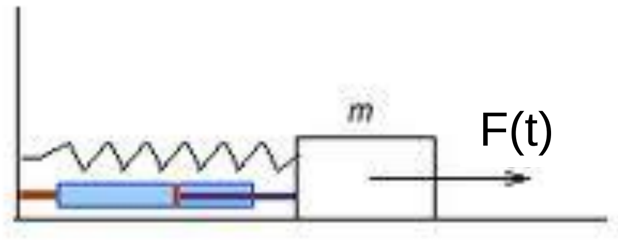


La ecuación que el sistema satisface es:

$$\ddot{x} + k_1\dot{x} + \omega_0^2 x = f_0 \sin(\omega t)$$

Calculamos las derivadas de primer y segundo orden

$$\dot{x}_P = \omega B_1 \cos(\omega t) - \omega B_2 \sin(\omega t)$$

$$\ddot{x}_P = -\omega^2 B_1 \sin(\omega t) - \omega^2 B_2 \cos(\omega t)$$

Remplazando en la ecuación resulta

$$\left[B_1\left(\omega_0^2 - \omega^2\right) + 2\alpha\omega B_2 - f_0\right]\sin(\omega t) + \left[2\alpha\omega B_1 + \left(\omega_0^2 - \omega^2\right)B_2\right]\cos(\omega t) = 0$$

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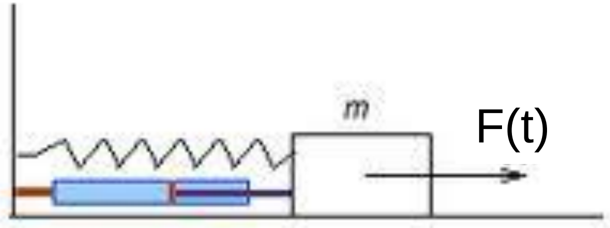
La ecuación que el sistema satisface es:




$$\ddot{x} + k_1 \dot{x} + \omega_0^2 x = f_0 \sin(\omega t)$$

$$x = x_H(t) + x_P(t)$$

$$\left(\omega_0^2 - \omega^2\right) B_1 + 2\alpha\omega B_2 = f_0$$

$$2\alpha\omega B_1 + \left(\omega_0^2 - \omega^2\right) B_2 = 0$$

$$\begin{bmatrix} \left(\omega_0^2 - \omega^2\right) & 2\alpha\omega \\ 2\alpha\omega & \left(\omega_0^2 - \omega^2\right) \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} f_0 \\ 0 \end{bmatrix}$$

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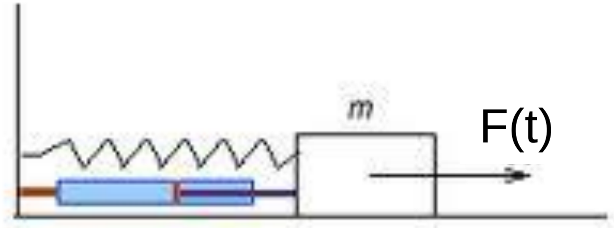


La ecuación que el sistema satisface es:

$$\ddot{x} + k_1\dot{x} + \omega_0^2 x = f_0 \sin(\omega t)$$

$$x = x_H(t) + x_P(t)$$

$$\left(\omega_0^2 - \omega^2\right)B_1 + 2\alpha\omega B_2 = f_0$$

$$2\alpha\omega B_1 + \left(\omega_0^2 - \omega^2\right)B_2 = 0$$

$$B_1 = \frac{f_0\left(\omega_0^2 - \omega^2\right)}{\left(\omega_0^2 - \omega^2\right)^2 + (2\alpha\omega)^2}$$

$$B_2 = \frac{f_0 2\alpha\omega}{\left(\omega_0^2 - \omega^2\right)^2 + (2\alpha\omega)^2}$$

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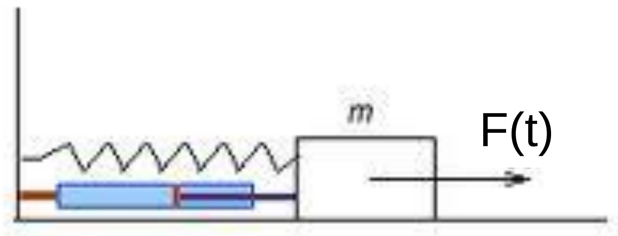


La ecuación que el sistema satisface es:

$$\left(\omega_0^2-\omega^2\right)B_1+2\alpha\,\omega\,B_2 = f_0$$
$$2\alpha\,\omega\,B_1+\left(\omega_0^2-\omega^2\right)B_2=0$$

$$\ddot{x}+k_1\dot{x}+\omega_0^2 x = f_0\sin(\omega t)$$

$$x = x_H(t)+x_P(t)$$

$$x_P = B_1\sin(\omega t)+B_2\cos(\omega t)$$

$$B^2 = B_1^2+B_2^2$$

$$x_P = B\sin(\omega t+\Phi) \quad donde$$

$$tg(\Phi)=\frac{B_2}{B_1}$$

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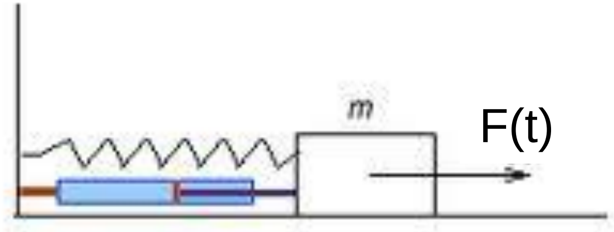


La ecuación que el sistema satisface es:

$$\ddot{x} + k_1 \dot{x} + \omega_0^2 x = f_0 \sin(\omega t)$$

$$x = x_H(t) + x_P(t)$$

$$\left(\omega_0^2 - \omega^2\right) B_1 + 2\alpha\omega B_2 = f_0$$
$$2\alpha\omega B_1 + \left(\omega_0^2 - \omega^2\right) B_2 = 0$$

$$x_P = B \sin(\omega t + \Phi)$$

$$B = \frac{f_0}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + (2\alpha\omega)^2}} \qquad tg(\Phi) = \frac{2\alpha\omega}{\omega_0^2 - \omega^2}$$

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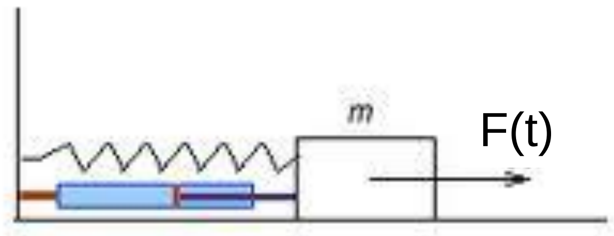


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$$tg(\Phi) = \frac{2\alpha\omega}{\omega_0^2 - \omega^2}$$

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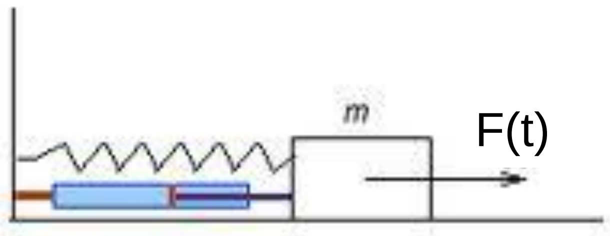


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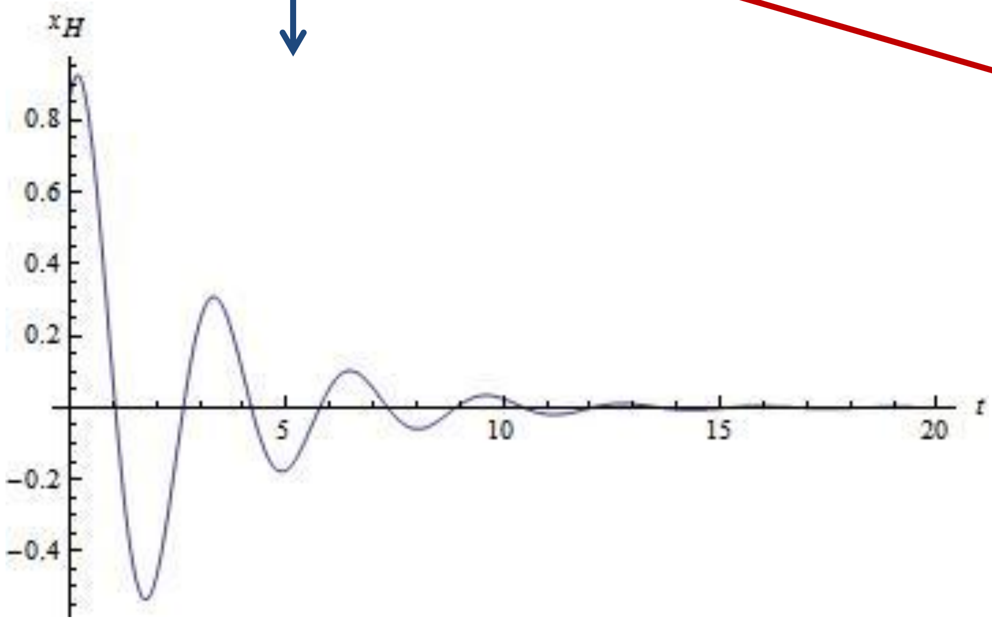


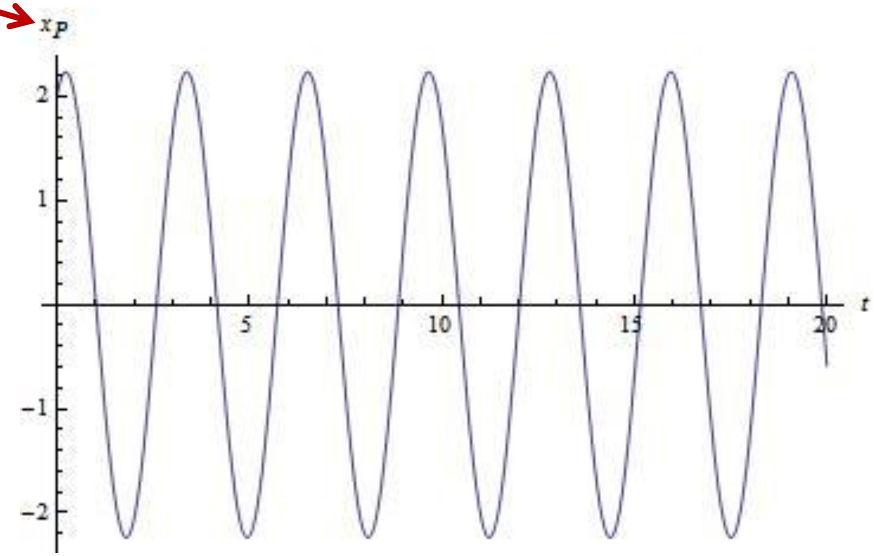

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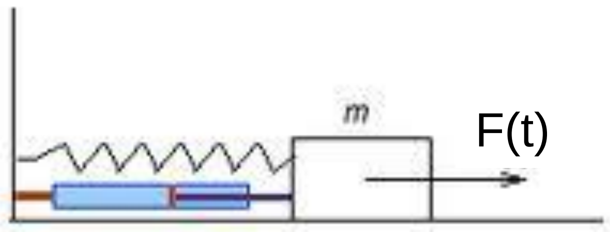


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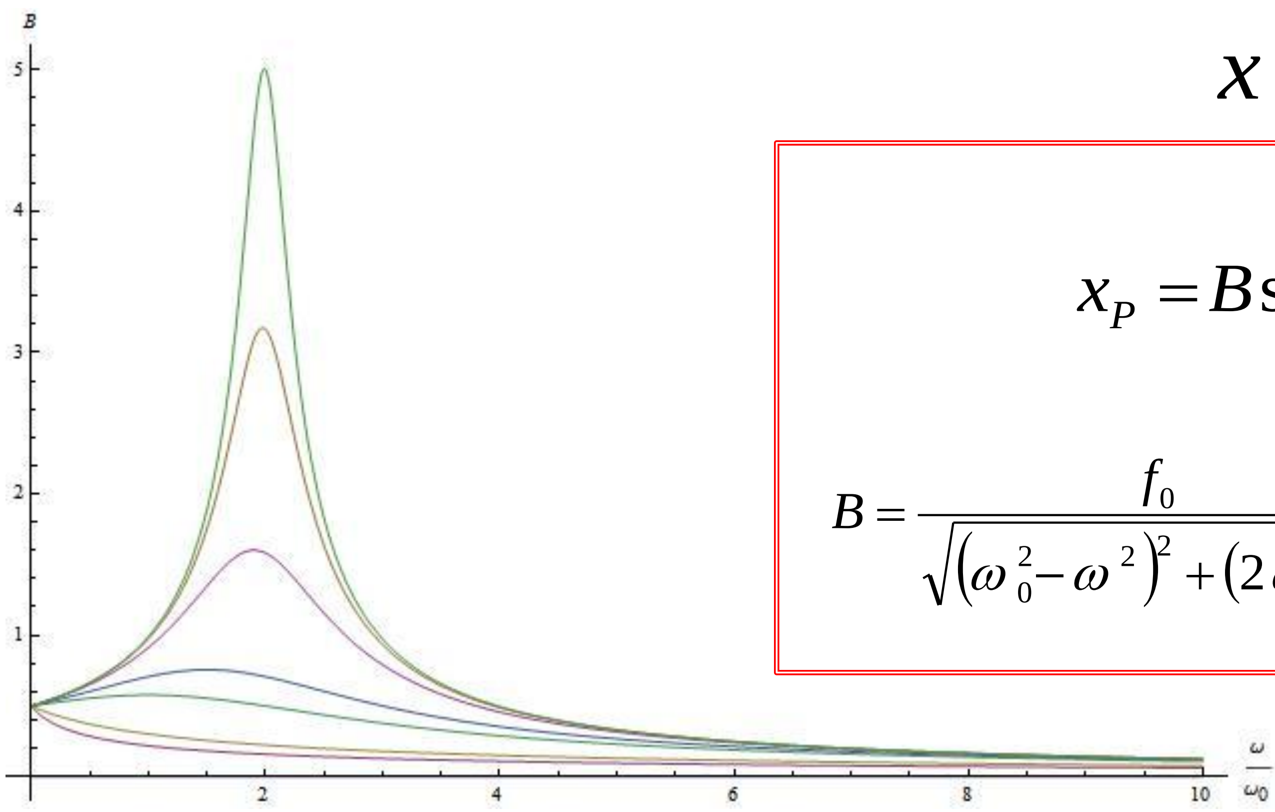
$$\ddot{x} + k_1\dot{x} + \omega_0^2 x = f_0 \sin(\omega t)$$




$$x = x_H(t) + x_P(t)$$

$$x_P = B\sin(\omega t + \Phi)$$

$$B = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\alpha\omega)^2}} \qquad tg(\Phi) = \frac{2\alpha\omega}{\omega_0^2 - \omega^2}$$

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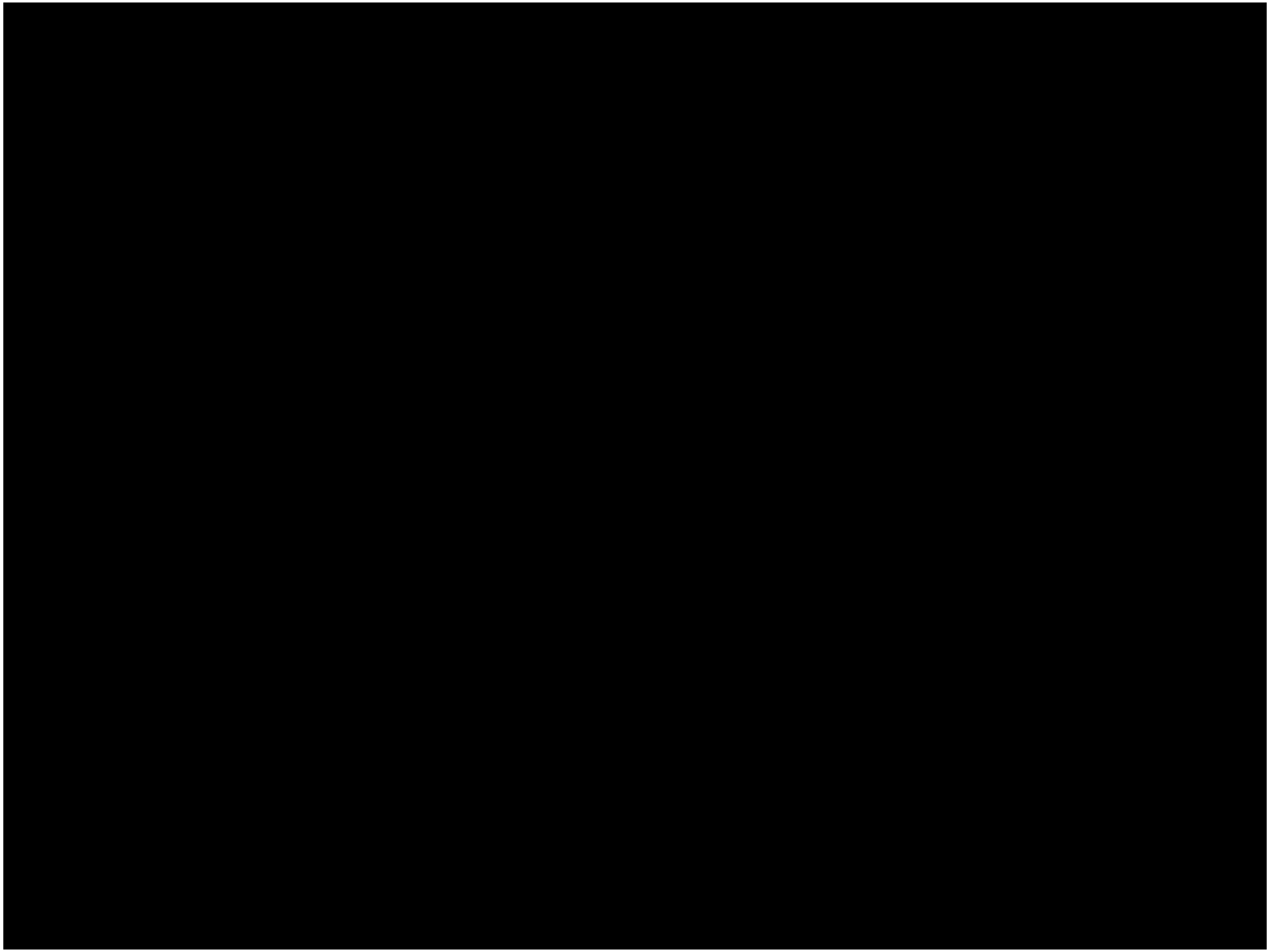

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http://www.youtube.com/watch?v=lFV6NBbjxME