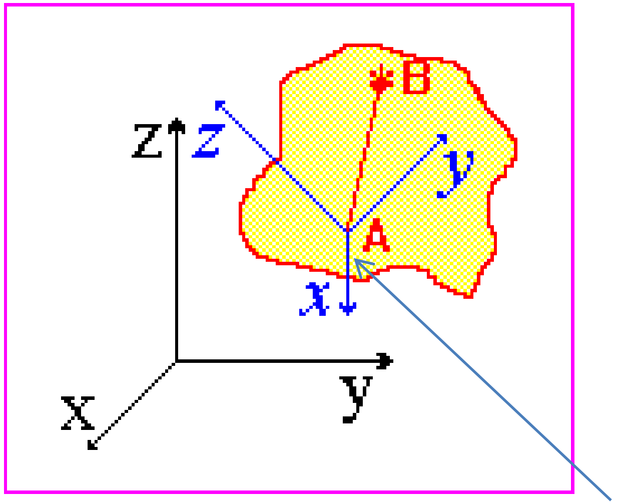
Cuerpos Rígidos

Cinemática

El cuerpo tiene **6 grados de libertad**

$$\vec{v}_B = \vec{v}_A + \vec{w} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{w} \times \vec{w} \times \vec{r}_{B/A} + \dot{\vec{w}} \times \vec{r}_{B/A}$$



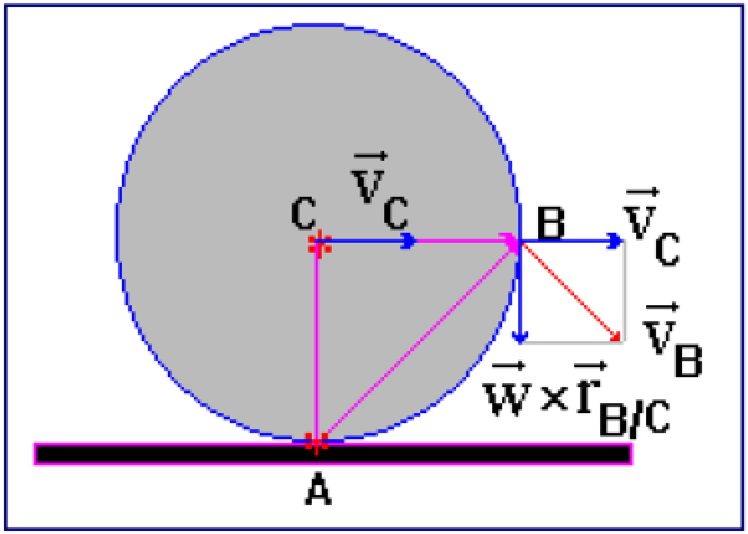
sistema auxiliar solidario al cuerpo

Cuerpos Rígidos

Cinemática

$$\vec{v}_C = \vec{w} \times \vec{r}_{C/A}$$
$$\vec{v}_B = \vec{w} \times \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_C + \vec{w} \times \vec{r}_{B/C}$$

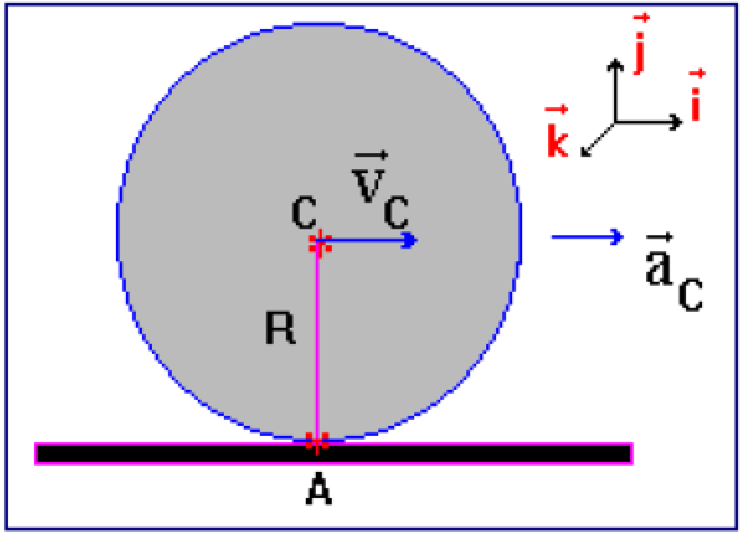


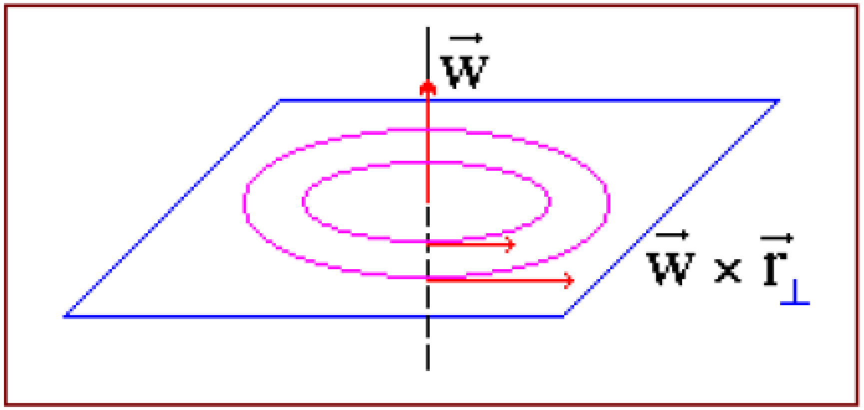
Cuerpos Rígidos

Movimiento plano

La dirección del eje de rotación permanece constante. Así, el vector aceleración angular debe cumplir la condición:

$$\dot{\vec{w}} \parallel \vec{w}$$

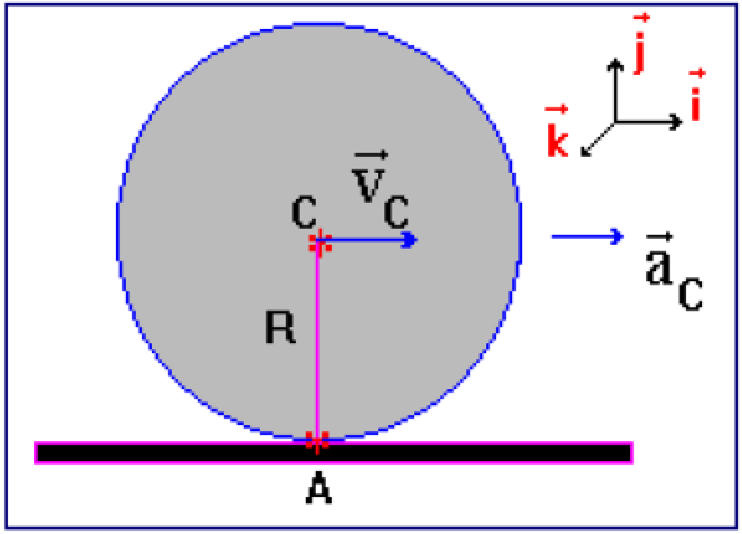




Cuerpos Rígidos

Movimiento plano

$$\vec{v}_C = \vec{w} \times \vec{r}_{C/A}$$



$$v_C = wR$$

$$w = \frac{v_C}{R}$$

$$\dot{w} = \frac{\dot{v}_C}{R}$$

$$\dot{\vec{w}} = -\frac{a_C}{R}\vec{k}$$

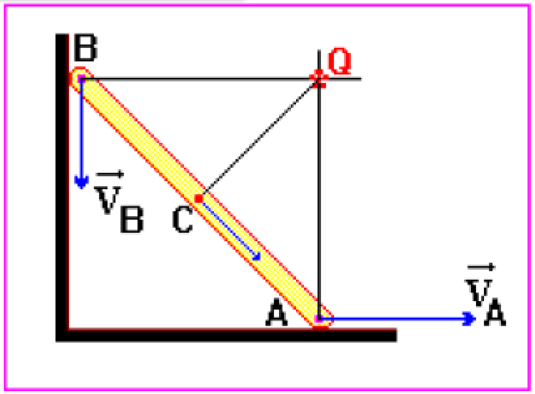
Cuerpos Rígidos

Centro de rotación instantánea

$$\vec{v}_Q = \vec{v}_A + \vec{w} \times \vec{r}_{Q/A}$$

$$\vec{v}_A + \vec{w} \times \vec{r}_{Q/A} = 0$$

$$\vec{v}_B = \vec{w} \times \vec{r}_{B/Q}$$

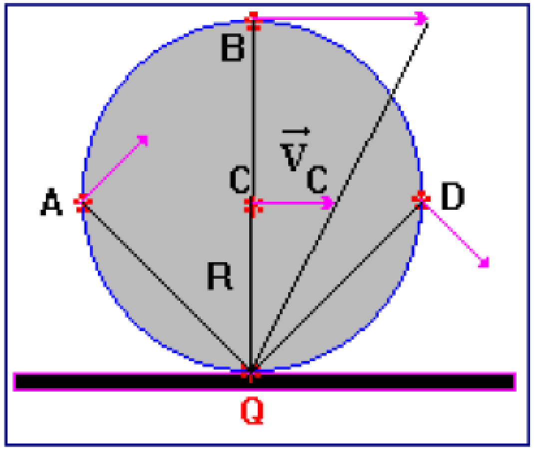


$$i \rightarrow v_x + wy = 0$$

$$j \rightarrow v_y - wx = 0$$

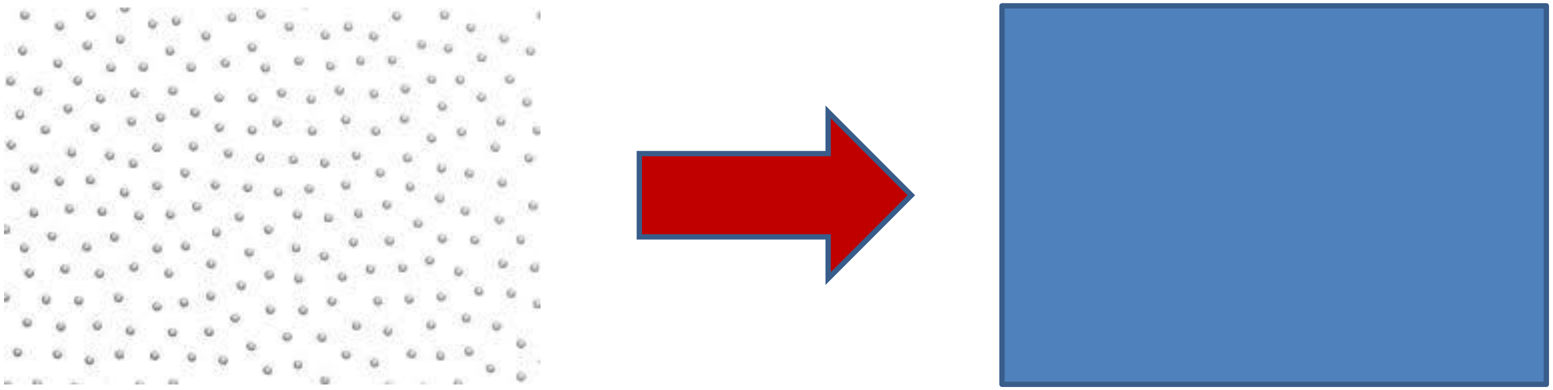
$$x = \frac{v_y}{w}$$

$$y = -\frac{v_x}{w}$$



Cuerpos Rígidos

Ecuaciones de movimiento



$$m_i \rightarrow dm = \rho(\vec{r})d\tau$$

$$\sum m_i \rightarrow \int_\tau \rho(\vec{r})d\tau$$

Sistema Rígido Discreto

$$\vec{r}_c = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

Sistema Rígido Continuo

$$\vec{r}_c = \frac{\int_\tau \vec{r}\rho(\vec{r})d\tau}{\int_\tau \rho(\vec{r})d\tau}$$

Cuerpos Rígidos

Ecuaciones de movimiento

Momento angular

$$\vec{L} = \vec{r}_c \times m\vec{v}_c + \vec{L}_c$$

$$\vec{L}_A = \vec{r}_{c/A} \times m\vec{v}_c + \vec{L}_c$$

$$\vec{L}_c = \sum \vec{r}_i' \times m_i \vec{v}_i' \qquad \vec{v}_i = \vec{v}_c + \vec{w} \times \vec{r}_i' \qquad \vec{L}_c = \sum m_i \vec{r}_i' \times (\vec{w} \times \vec{r}_i')$$

$$\vec{L}_c = \sum m_i \left| \vec{w}\, r_i^2 - \vec{r}_i (\vec{w} \cdot \vec{r}_i) \right|$$

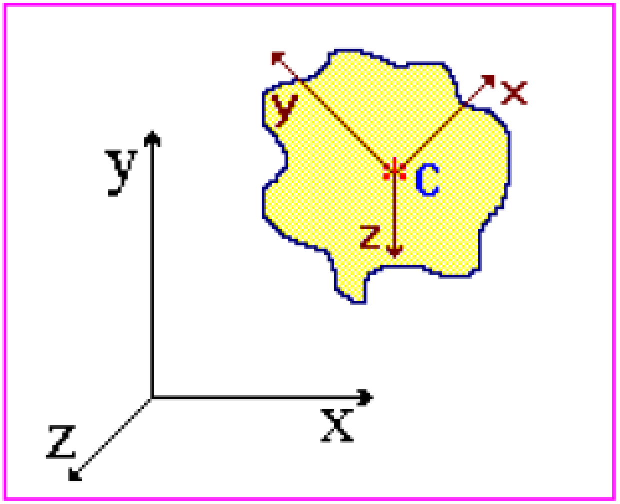
Cuerpos Rígidos

Ecuaciones de movimiento

Momento angular

$$\vec{L}_c = \sum m_i \left[\vec{w}\, r_i^2 - \vec{r}_i(\vec{w}\cdot\vec{r}_i)\right]$$

$$\vec{w} = w\,\vec{k}$$



$$\vec{L}_c = \sum m_i \left[(x_i^2 + y_i^2 + z_i^2) w\,\vec{k} - (x_i\vec{i} + y_i\vec{j} + z_i\vec{k}) w z_i\right]$$

$$\vec{L}_c = \left[-\sum m_i x_i z_i\right] w\vec{i} + \left[-\sum m_i y_i z_i\right] w\vec{j} + \left[\sum m_i (x_i^2 + y_i^2)\right] w\vec{k}$$

Cuerpos Rígidos

Ecuaciones de movimiento

Momento angular

$$\vec{L}_c = \left[\sum m_i (x_i^2 + y_i^2)\right] w\,\vec{k}$$

$$\vec{L}_c = I_c \vec{w}$$

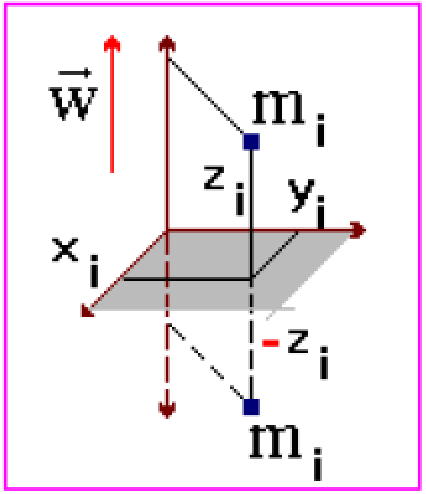
$$I_c = \int_\tau (x_i^2 + y_i^2)\rho(xyz)\,d\tau$$

$$\vec{L} = \vec{r}_c \times m v_c + I_c \vec{w}$$

$$\vec{L}_A = \vec{r}_{c/A} \times m v_c + I_c \vec{w}$$

$$\vec{L}_Q = \sum \vec{r}_i \times m_i \vec{v}_i$$

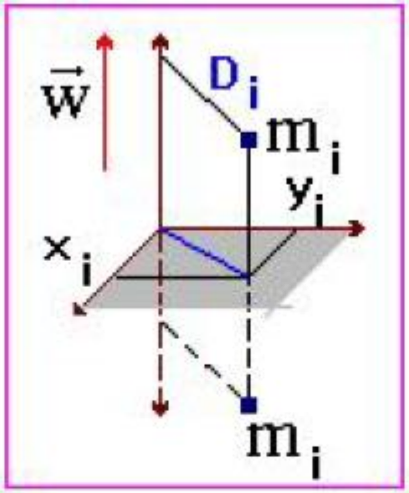
$$\vec{L}_Q = I_Q \vec{w}$$



$$I_c = \sum m_i (x_i^2 + y_i^2)$$

$$D_i^2 = x_i^2 + y_i^2$$

$$I_c = \sum m_i D_i^2$$



Cuerpos Rígidos

Ecuaciones de movimiento

Ecuación de Newton

$$\vec{F} = m\vec{a}_c \qquad \vec{F} = \left.\frac{d\vec{P}}{dt}\right|_{XYZ} \qquad \vec{F} = \left.\frac{d\vec{P}}{dt}\right|_{xyz} + \vec{w} \times \vec{P}$$

Cuerpos Rígidos

Ecuaciones de movimiento

Ecuaciones de Momento

$$\vec{M}_A = \left.\frac{d\vec{L}_A}{dt}\right|_{XYZ} + \vec{v}_A \times m\vec{v}_c$$

$$\vec{M}_c = \left.\frac{d\vec{L}_c}{dt}\right|_{XYZ}$$

$$\vec{M}_c = \left.\frac{d\vec{L}_c}{dt}\right|_{xyz} + \vec{w} \times \vec{L}_c$$

$$\vec{M}_c = I_c \left.\frac{d\vec{w}}{dt}\right|_{xyz} + \vec{w} \times I_c \vec{w}$$

$$\vec{M}_c = I_c \dot{\vec{w}}$$

$$\vec{M}_c = I_c \vec{\alpha}$$

$$\vec{M}_Q = \left.\frac{d\vec{L}_Q}{dt}\right|_{XYZ}$$

$$\vec{M}_Q = \left.\frac{d\vec{L}_Q}{dt}\right|_{xyz} + \vec{w} \times \vec{L}_Q$$

$$\vec{M}_Q = I_Q \left.\frac{d\vec{w}}{dt}\right|_{xyz} + \vec{w} \times I_Q \vec{w}$$

$$\vec{M}_Q = I_Q \dot{\vec{w}}$$

$$\vec{M}_Q = I_Q \vec{\alpha}$$

Cuerpos Rígidos

Ecuaciones de movimiento

$$\vec{F} = m\vec{a}_c$$

$$\vec{F} = \left.\frac{d\vec{P}}{dt}\right|_{XYZ}$$

$$\vec{F} = \left.\frac{d\vec{P}}{dt}\right|_{xyz} + \vec{w} \times \vec{P}$$

$$\vec{M}_A = \left.\frac{d\vec{L}_A}{dt}\right|_{XYZ} + \vec{v}_A \times m\vec{v}_c$$

$$\vec{M}_c = I_c\vec{\alpha}$$

$$\vec{M}_Q = I_Q\vec{\alpha}$$