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Citation: Am. J. Phys. **56**, 138 (1988); doi: 10.1119/1.15692
View online: http://dx.doi.org/10.1119/1.15692
View Table of Contents: http://ajp.aapt.org/resource/1/AJPIAS/v56/i2
Published by the American Association of Physics Teachers

The aerodynamics of tennis balls—The topspin lob

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(Received 1 May 1986; accepted for publication 30 April 1987)

A general description is presented of the calculation of the ballistic trajectory of a flying spinning ball acted on, in addition to the forces of gravity and drag, by the so-called Magnus force. By applying the regression analysis to results of wind-tunnel measurement of the drag and lift coefficients of a spinning ball, a calculation of the nonlinear differential equation of the hodograph was carried out by means of the Runge–Kutta method. The theoretical results that can be used to calculate the ballistic trajectories for any ball game were applied to one of the most difficult and most interesting tennis strokes, i.e., to the topspin lob. Practical results obtained for various distances are presented in a table as well as in graphical form.

LIST OF SYMBOLS

v	ball flight velocity
w	equatorial velocity of the spinning ball
C_D	drag coefficient
C_L	lift coefficient
R	ballistic trajectory radius of curvature
m	ball mass
G	ball weight
g	gravity acceleration
τ	angle between the velocity vector and a horizontal plane
α	initial stroke angle
s	length of the ballistic trajectory
ρ	air density
D	drag force
M	Magnus force
D^*, M^*	dimensionless drag and Magnus force
x, y	Cartesian coordinate of the ballistic trajectory
y_0	initial ball height
Re	Reynolds number
n	ball revolutions

I. INTRODUCTION AND HISTORICAL BACKGROUND

If the Coriolis force is neglected, the shape of the ballistic trajectory of a flying rotating ball is essentially affected by three forces, i.e., the force of gravity G, the drag force D, and the so-called Magnus force M, which was explained by Magnus as early as 1853.[1] This force always acts perpendicular to the vector of the flying ball velocity and its axis of rotation. Its magnitude and, in particular, the direction of the ball rotation substantially affects the shape of the ball's trajectory in many games such as tennis, golf, ping-pong, baseball, etc.

A considerable amount of literature dealing with the force effects on a rotating cylinder has been published over the years; experimental studies of rotating spheres, on the other hand, have been available in a limited number only, e.g., Maccoll[2] and Höerner[3] measured the force effects on a rotating smooth sphere and Davis[4] those acting on golf balls with various surfaces. A feature common to these studies was the determination of C_D and C_L, the drag and lift coefficient characteristics of a rotating sphere.

In contrast to these studies and their quantitative results, one finds in numerous publications only qualitative results and an analysis of the reason of curving of a spinning ball trajectory made on the basis of simple considerations founded on the Bernoulli equation; see, for example, Refs. 5 and 6.

II. EQUATION GOVERNING THE TRAJECTORY OF FLIGHT—THEORETICAL INTRODUCTION

Consider the case of a rotating ball projected with an initial velocity v_0 at an angle α, the axis of rotation of which is parallel to the horizontal plane. In the case of a lifted or topspin stroke (lob), the direction of the rotation is such that the vector of the angular velocity $\mathbf{\Omega}$, when following the flying ball, lies along an axis parallel to the horizontal plane and aims from right to left. The Magnus force acts towards the center of curvature of the ballistic trajectory thus increasing the trajectory curving and shortening the range compared with the trajectory of the nonrotating ball.

The calculation of the flight trajectory starts from the equilibrium of forces into the normal (Fig. 1),

$$mv^2/R = mg\cos\tau + M. \tag{1}$$

Noting that τ decreases as the arc length s increases we have $R = -ds/d\tau$, recalling that $dt = ds/v$, $dx = ds\cos\tau$, and $dy = ds\sin\tau$, one obtains after elimination of R from Eq. (1) and integration, the parametric equation of the ballistic trajectory in the form

$$x = -\frac{1}{g}\int_\alpha^\tau \frac{v^2\cos\tau}{\cos\tau + M^*}\,d\tau, \tag{2}$$

$$y = y_0 - \frac{1}{g}\int_\alpha^\tau \frac{v^2\sin\tau}{\cos\tau + M^*}\,d\tau. \tag{3}$$

The time from instant of striking necessary to reach these coordinates is obtained by the expression

$$t = -\frac{1}{g}\int_\alpha^\tau \frac{v}{\cos\tau + M^*}\,d\tau, \tag{4}$$

where $M^* = M/mg$ is dimensionless Magnus force referred to the unit weight of the ball. To be in position to carry out the integration in Eqs. (2)–(4) one must first determine the dependence of the ball velocity on angle τ, i.e., write the equation of the hodograph $v = v(\tau)$. One proceeds from the equation of motion that, for the tangen-

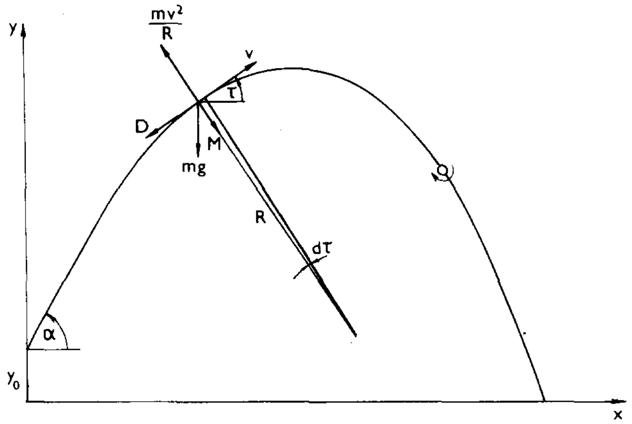

Fig. 1. Ballistic trajectory and forces acting on a flying and rotating ball.

tial direction at a point of the ballistic trajectory (Fig. 1), has the form

$$m \frac{dv}{dt} = -D - mg \sin \tau. \tag{5}$$

Substituting for dt in Eq. (5) and some manipulation leads to the differential equation of the hodograph in the form

$$\frac{dv}{d\tau} = \frac{\sin \tau + D^*}{\cos \tau + M^*} v, \tag{6}$$

where similarly $D^* = D/mg$ is the dimensionless drag force referred to the unit weight of the ball. For the case $M^* = 0$, Eq. (6) corresponds to the usually used hodograph; see, e.g., Ref. 7.

A dimensional analysis, e.g., Ref. 8, in the case of a flying ball leads to the conclusion that the dimensionless drag and Magnus force turn out to be

$$D^* = C_D (\pi d^2/8mg)\, \rho v^2 \tag{7}$$

$$M^* = C_L (\pi d^2/8mg)\, \rho v^2, \tag{8}$$

where d is the ball diameter and ρ is the air density. The drag and lift coefficients C_D and C_L for a spinning ball can be considered $C_D = f(w/v, \mathrm{Re})$, $C_L = f(w/v, \mathrm{Re})$, where w is the equatorial velocity of a flying ball and Re is the Reynolds number. The effect of the Mach number is practically negligible up to $\mathrm{Ma} < 0.3$.

III. MEASUREMENT OF THE LIFT AND DRAG COEFFICIENTS

A device for ejecting spinning balls in the aerodynamic tunnel is shown in Fig. 2. In the experiments, balls of the Tretorn trademark were used. These balls, manufactured pressureless, were fastened into a special fixture, after which coaxial holes were drilled through the ball. Into the holes, miniature brass bearings were glued in such a way as to enable the ball to be fixed between steel pointed centers. In comparison with other, mostly pressurized, balls, its properties remain unchanged.

Ball spinning was initiated through a small electric motor with a foam rubber conical follower that was pressed against the ball surface. After releasing the pin that was pressing the electric drive into engagement with the ball, a trigger was synchronously released, which in turn immediately detracted, through wires, the steel centers allowing the ball to fall freely into the air stream of the tunnel.

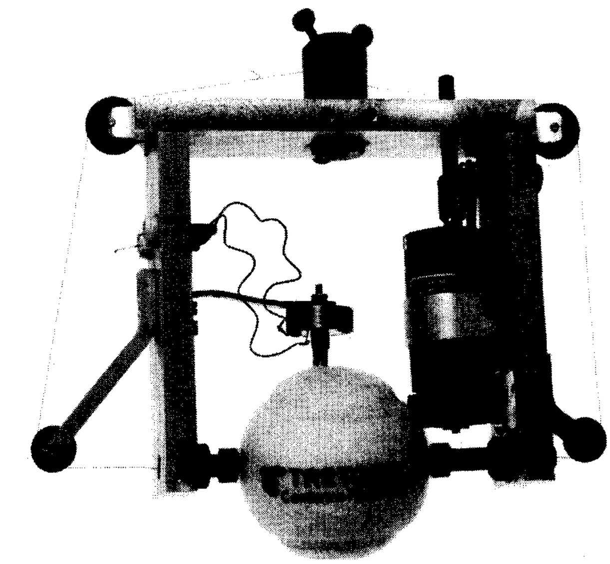

Fig. 2. Device for rotating and releasing the ball.

Spinning speed of the ball was measured by an induction sensor located closely above the ball, see Fig. 2, picking up signals from a small ferrite magnet, glued into a hole drilled on the ball periphery. Simultaneously, the spinning speed was measured by a stroboscope. Since a complete agreement between both values was found to exist in the whole

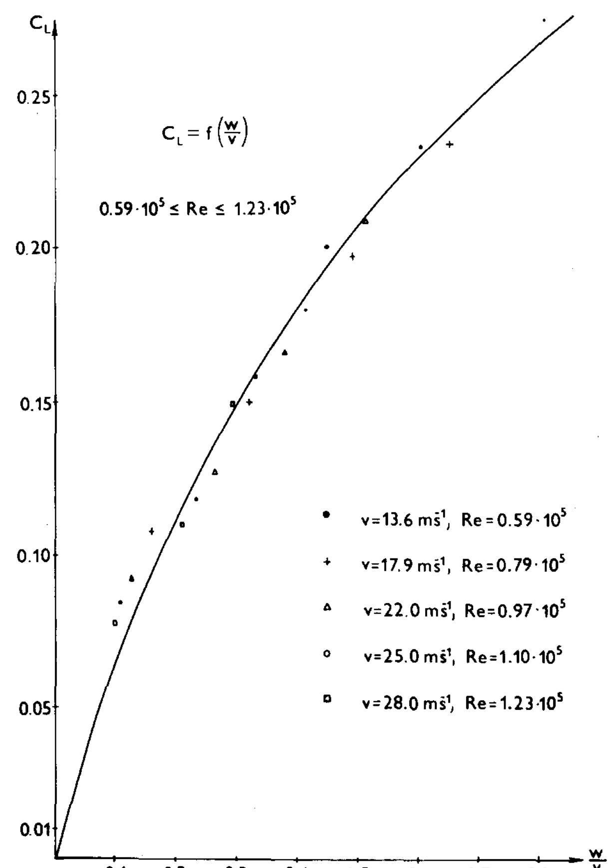

Fig. 3. Measured values of the lift coefficient C_L for various air stream velocities. The resulting regression curve applies for all data.

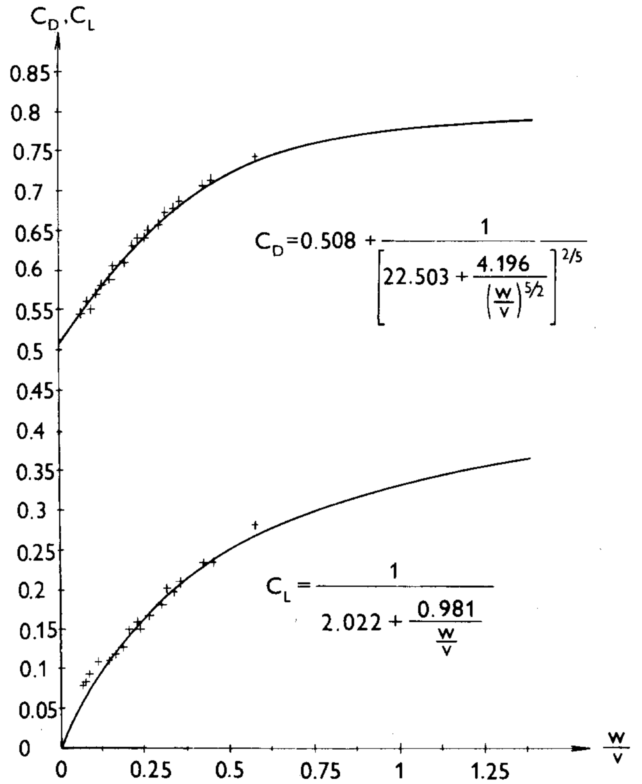

Fig. 4. Values of the drag and lift coefficient as a function of the w/v ratio.

range of the spinning speed used in the experiments, the induction sensor was removed, thus ensuring a more realistic flow pattern around the ball, especially in the initial stages of each experimental run.

Measurements were carried out in the open-type aerodynamics tunnel of the Research Institute for Aeronautics in Prague, which has a diameter of 1.8 m. The drag and lift coefficients C_D and C_L can be determined using a simple procedure described, e.g., in Ref. 4. Measurements were made at air velocities $13.6 \leqslant v \leqslant 28 \text{ m s}^{-1}$ and ball revolution $800 \leqslant n \leqslant 3250$ rpm. Measured values of the coefficients were plotted in the form of $C_D = C_D(w/v)$ and $C_L = C_L(w/v)$ relations in which the Reynolds number, in accordance with dimensional considerations, is a parameter. However, results of measurements in the aerodynamics tunnel for the above-mentioned range of air velocities and revolution were so scattered, see Fig. 3 (similar scatter was obtained for the C_D value) that a distinct dependence upon the air velocity, and thus upon Re, could not be obtained. Therefore we can conclude that, within the limits of measurement precision, the dependence of C_D and C_L upon Re may be neglected, which greatly simplifies the forthcoming calculation.

Therefore, in order to smooth the measured values, it was possible to make use of a one-dimensional regression function whose general form was chosen to be $y = (a_0 + a_1 x^{k_1} + a_2 x^{k_2})^{k_3}$, see Ref. 9. Using a nonlinear regression, employing the method of least squares, 50 curves were obtained in which the standard deviation $s(y_i)$ was chosen as the criterion for a best fit. Using this procedure, the following equations for the C_D and C_L coefficients were obtained, see Fig. 4:

$$C_D = 0.508 + \frac{1}{[22.503 + 4.196(w/v)^{-5/2}]^{2/5}}, \qquad (9)$$

$$C_L = \frac{1}{2.202 + 0.981(w/v)^{-1}}, \qquad (10)$$

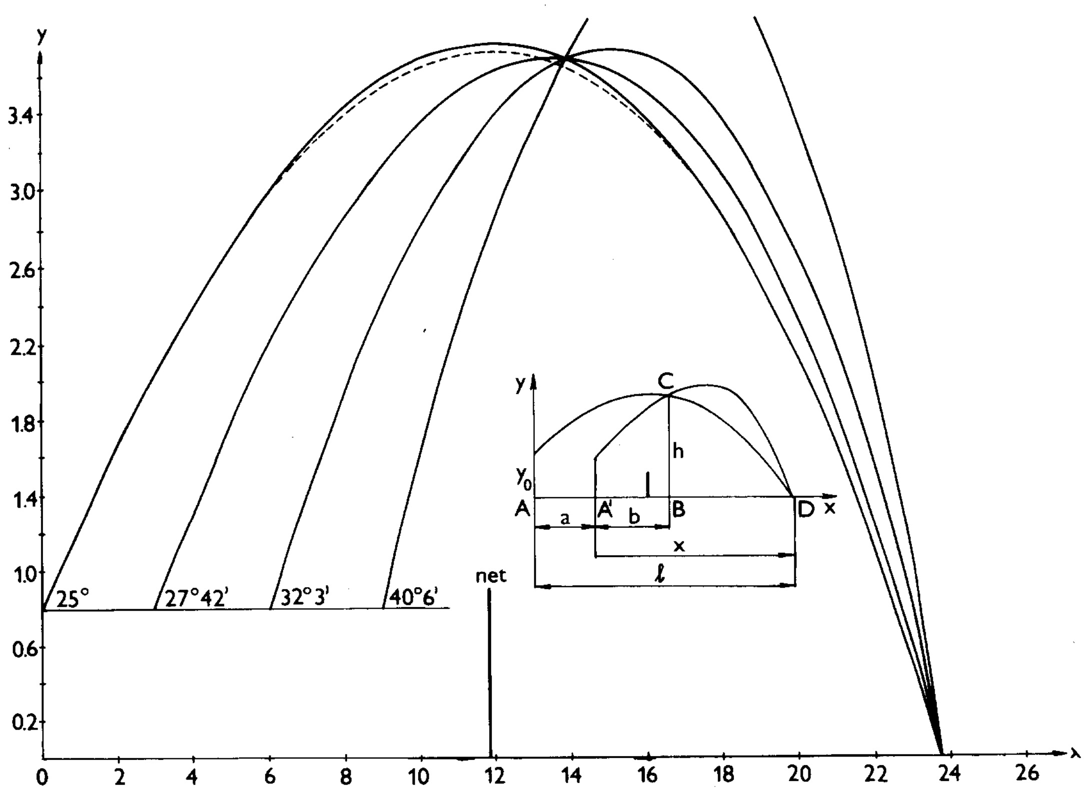

Fig. 5. Calculated ball trajectories for various values of a and b with $l = 23.77$ m, $y_0 = 0.8$ m, and $H = 3.7$ m.

Table I. Numerical results of the mathematical model for $l = 23.77$ m, initial height $y_0 = 0.8$ m, and obstacle height $H = 3.7$ m.

a	b	α	n	t_C	t_{max}	v_0	w/v	$Re \times 10^{-5}$
			0	0.873	1.672	19.35	0	0.87–0.59
0	13.885	25°	2500	0.751	1.480	23.00	0.37–0.63	1.04–0.62
			3500	0.729	1.440	23.75	0.51–0.87	1.07–0.63
			0	0.776	1.668	17.19	0	0.78–0.54
3	10.885	27°42″	2500	0.675	1.505	20.14	0.43–0.70	0.91–0.56
			3500	0.661	1.474	20.59	0.59–0.96	0.93–0.57
			0	0.654	1.680	15.21	0	0.69–0.47
6	7.885	32°3′	2500	0.581	1.551	17.31	0.50–0.79	0.78–0.49
			3500	0.572	1.531	17.61	0.69–1.09	0.80–0.50
			0	0.511	1.745	13.26	0	0.60–0.38
			2500	0.462	1.654	14.71	0.59–0.96	0.66–0.40
9	4.885	41°6′	3500	0.458	1.642	14.81	0.82–1.34	0.67–0.41
			4500	0.457	1.636	14.86	1.05–1.71	0.67–0.41
			6000	0.455	1.633	14.91	1.39–2.28	0.68–0.41

with standard deviations of $s(C_D) = 0.086$ and $s(C_L) = 0.11$. Equations (9) and (10) are very similar to the course of relationships obtained by Davies[4] for dimple and mesh golf balls. These relations also have their limiting values $(C_D)_{lim} = 0.796$ and $(C_L)_{lim} = 0.494$.

IV. NUMERICAL CALCULATION

As a first step, the differential equation of the hodograph (6) has been solved, using the Runge–Kutta method and the initial conditions $\tau = \alpha$, $v = v_0$. The starting value

$$v_0 = \sqrt{\frac{(g/2)x_{max}^2(1 + \tan^2\alpha)}{y_0 + x_{max}\tan\alpha}} \tag{11}$$

corresponds to a velocity at which the ball would travel, from an initial height y_0 and an initial projection angle α, a distance x_{max} in a vacuum. On the basis of discrete velocity value calculated from Eq. (6), it is not difficult to determine, through numerical integration, coordinates of the ballistic trajectory from Eqs. (2) and (3) and the corresponding time necessary for traveling to that point from Eq. (4). It is clear that using Eq. (6), respecting variable air resistance and trajectory curvature due to the Magnus force, the ball cannot reach the distance x_{max}. Therefore, in subsequent calculations, the initial velocity v_0 is increased by an increment $\Delta v = 1$ m s^{-1} till the required value x_{max} was exceeded. After that, velocity was diminished by one step and a finer value $\Delta v/10$ introduced into the algorithm. The whole procedure was repeated till the prescribed x_{max} was reached with a preset accuracy. In Table I, corresponding to Fig. 5, results of the above procedure are summarized as an illustration. Here, the calculations were made for one of the most difficult and interesting tennis strokes, i.e., for the topspin lob. If the length of the tennis court is 23.77 m and, while playing doubles, the opposing netman stands 2 m behind the net (see point B in Fig. 5), it is possible to determine from Table I, for a given distance of the striking A player, the initial angle α and initial velocity v_0 for a given spin, which for an obstacle of the height h (e.g., $h = 3.7$ m) would result in a most rapid stroke falling just upon the opposer's baseline, point D.

In the computer program, the input values are distances a and b, court length l, height h, revolutions of a ball n, and the initial height y_0. For these input values the program determines the angle α, the velocity v_0, the total flight time t_{max}, and the important value of time necessary to reach the point C, t_C. Moreover, Table I presents ranges of the w/v ratio and the Reynolds number Re as a check. It is clear that on increasing the ball spin, the time necessary to reach the critical point C decreases steadily so that the player must react more and more rapidly. If, e.g., a player intends to lob the opposing netman from the baseline, $b = 13.885$ m, the time without rotation is $t_C = 0.873$ s. However, playing a lifted stroke with $n = 3500$ rpm, t_C decreases to 0.729 s. This means that the opposer's reaction must be approximately 0.14 s more rapid. The effect of the topspin lob becomes even more pronounced, if played from somewhere within the court, e.g., from point A′. In this case the ball must be played under a greater angle. If, e.g., $a = 9$ m and $b = 4.88$ m, one can see from Table I that for $n = 3500$ rpm we must react at the latest in 0.46 s, regardless of the fact that the ball ascends steeply with the angle $\alpha = 40°6'$ so that it appears visually to fall well behind the baseline. This fact, together with the need for an extraordinarily fast reaction, influences in the initial stage of the ball flight, the decision making of even experienced players. It happens thus that the opposing player does not react properly to the ball's flight in spite of its reaching a lower height than in C, at which it could be returned.

It is interesting to note that for a constant distance b, the total flight distance being the same, all ballistic trajectories are approximately the same. They differ insignificantly in the vicinity of their maxima where all trajectories for rotating balls are below the ballistic curve without rotation, i.e., for $n = 0$.

In Fig. 5, for $a = 0$, $b = 13.88$ m, the dashed line corresponds to a ball rotating with $n = 3500$ rpm. The maximum of this trajectory lies only 4.1 cm below the curve for $n = 0$. The difference diminishes, attaining for $a = 9$ m only 1.4 cm.

For other values of l, h, a, b, n, and y_0, that might be of interest, a FORTRAN IV program is available and may be obtained from the author on request.

V. COURT—EXPERIMENT

In order to obtain real values of the tennis-ball rotational speed for a topspin lob played optimally, a high-speed camera STALEX with a maximum frequency of 3000 frames per second was employed. In shooting the movie, Czechoslovak Davis Cup player and doubles specialist Pavel Složil played this difficult stroke repeatedly with an effort to achieve a maximum possible spin.

After evaluating all the film material it became clear that the highest rotation obtained was around 3500 rpm. Although this is probably not the final limit of human possibilities these days, it is sufficient to play a fast and effective lob stroke. Therefore, the limiting value of $n = 3500$ rpm also closes the fourth column in Table I. Only for $a = 9$ m, as an illustrative example, the calculation was made for higher spin values of $n = 4500$ rpm and $n = 6000$ rpm. It may be seen that increasing the spin further above 3500 rpm results in accelerating the ball into point C by 0.001 s or by 0.003 s at 6000 rpm. From both viewpoints, i.e., what is practical and possible, it is clear that this insignificant acceleration does not produce any appreciable time gain for the attacking (or defending) player. On the other hand, it can only play a significant role after contacting the playground where it causes the ball to bounce off fast and high with a higher spin requiring more skill returning the ball.

ACKNOWLEDGMENTS

The author is indebted to Pavel Složil and to Jiří Valta, Director of the Tennis Center in Prague, for their generous help in shooting the movie mentioned in the experimental part of the article. I am also very grateful to my son Tony for continuous assistance and to Z. Kober from the Research Institute for Agriculture for excellent and professional cameraman's work.

Further, my thanks are due to E. Bornhorst and Dr. M. Jirsák, from the Research Institute for Aeronautics in Prague for their help in making experiments in the aerodynamic tunnel. Last but not least I should like to thank C. Suk, chairman of the Czechoslovak Tennis Association, Dr. Vl. Šafařík, Head of the Tennis Department on the Institute of Sports in Prague, and the professional photographer Pavel Štecha for the documentation and the photography.

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Teaching special relativity through a computer conference

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(Received 23 February 1987; accepted for publication 7 May 1987)

A recent seminar in special relativity is described, which was taught exclusively through a computer conference, hosted on a distant mainframe computer, and asynchronously accessed by students and instructor with microcomputer and modem. Nine participants offered more than 400 separate discussion contributions over the 13-week span of the course. Criteria for choosing courses to be offered in this mode are suggested, and problem areas that need attention in the conduct of subsequent courses are pointed out.

I. INTRODUCTION

In a recent article, Halloun and Hestenes[1] addressed the perils we face in the physics classroom if we ignore the fact that our students have preconceived, and often incorrect, notions of how nature behaves. A major part of our task as teachers is to address and correct these erroneous ideas. The "common sense" test described in Ref. 1 shows the extent of the problem and indicates that merely knowing how to calculate the motion of a projectile is not the same as knowing what it does, an important point also recently addressed by Gerhart.[2]

Many, perhaps most, of our physics classes depend almost exclusively on calculation and symbol manipulation, with little opportunity to discuss physics in plain language terms, ones that are rooted in our experience.[3] It seems reasonable to test the idea that successful learning of physics requires the use of word symbols as well as mathematical symbols, and thus we offered our required "Special Topics in Physics" course in a new mode that would require extensive text-based discussion.

The tool for this offering appeared at the same time, namely, the computer conference, in which all contributions to a discussion are made in words or at least entered on a typewriter keyboard. The use of a computer conference allowed us also to test the proposition that college

