DIFFERENTIAL OPERATORS

Electromagnetismo I - UNS

Cartesian Coordinates

Divergence

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Gradient

$$\nabla f = \frac{\partial f}{\partial x}\,\breve{\mathbf{x}} + \frac{\partial f}{\partial y}\,\breve{\mathbf{y}} + \frac{\partial f}{\partial z}\,\breve{\mathbf{z}}$$

Curl

$$(\nabla \times \mathbf{A})_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$$
$$(\nabla \times \mathbf{A})_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}$$
$$(\nabla \times \mathbf{A})_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical Coordinates

Divergence

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Gradient

$$\nabla f = \frac{\partial f}{\partial r}\,\breve{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\,\breve{\phi} + \frac{\partial f}{\partial z}\,\breve{\mathbf{z}}$$

Curl

$$(\nabla \times \mathbf{A})_r = \frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}$$

$$(\nabla \times \mathbf{A})_\phi = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}$$

$$(\nabla \times \mathbf{A})_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) - \frac{1}{r}\frac{\partial A_r}{\partial \phi}$$

Laplacian

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates

Divergence

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Gradient

$$\nabla f = \frac{\partial f}{\partial r}\,\breve{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\breve{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\,\breve{\phi}$$

Curl

$$(\nabla \times \mathbf{A})_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \phi}$$

$$(\nabla \times \mathbf{A})_\theta = \frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)$$

$$(\nabla \times \mathbf{A})_\phi = \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta}$$

Laplacian

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$