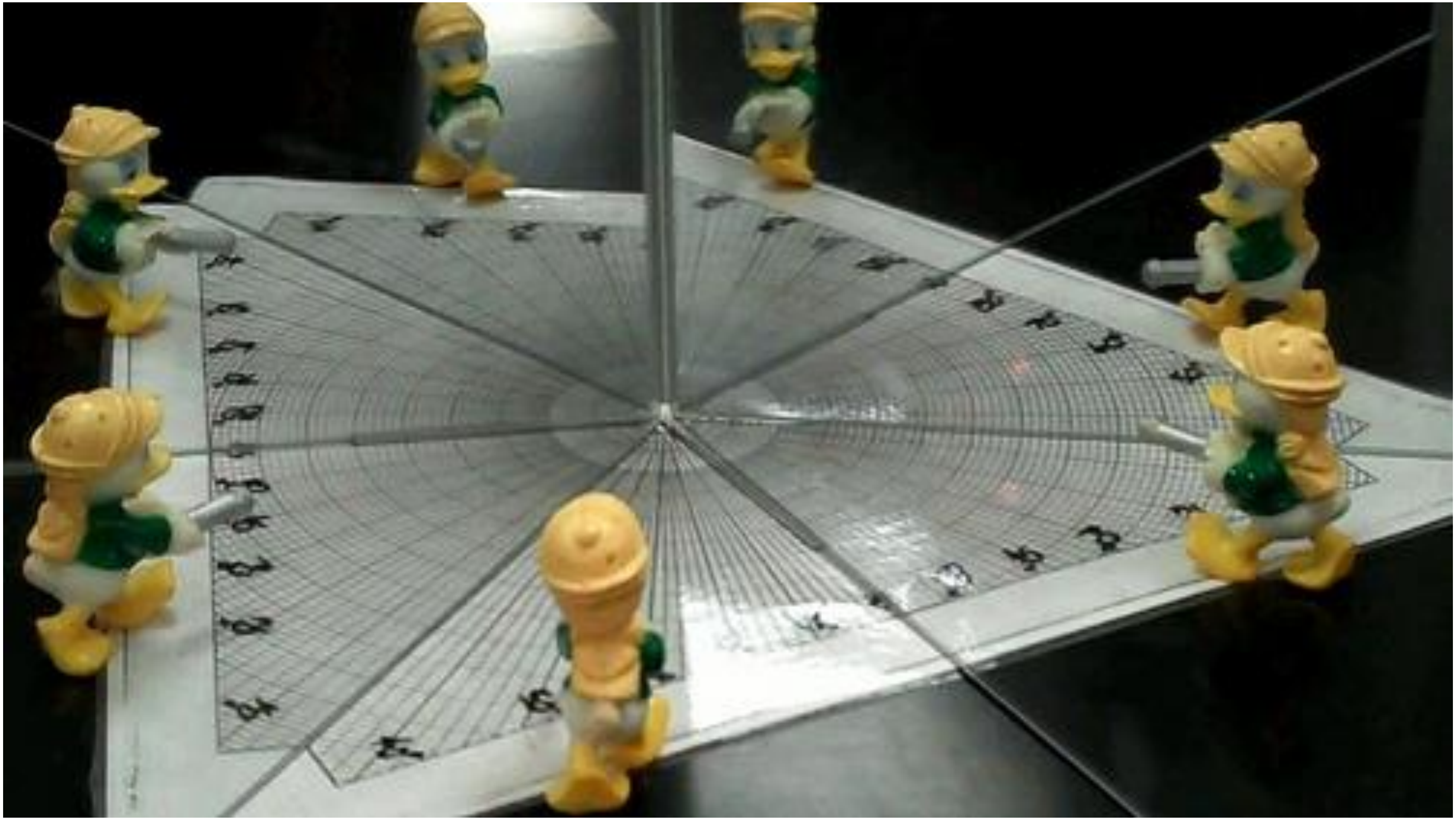
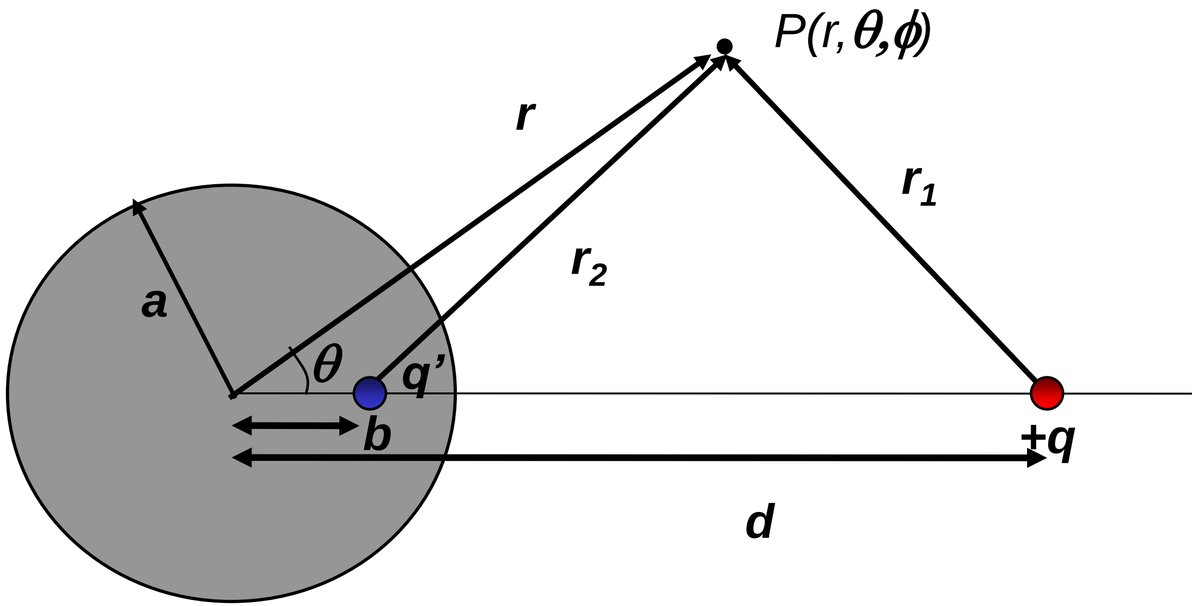
Electricidad



Universidad Nacional del Sur, Septiembre de 2019

Repaso - El método de las imágenes

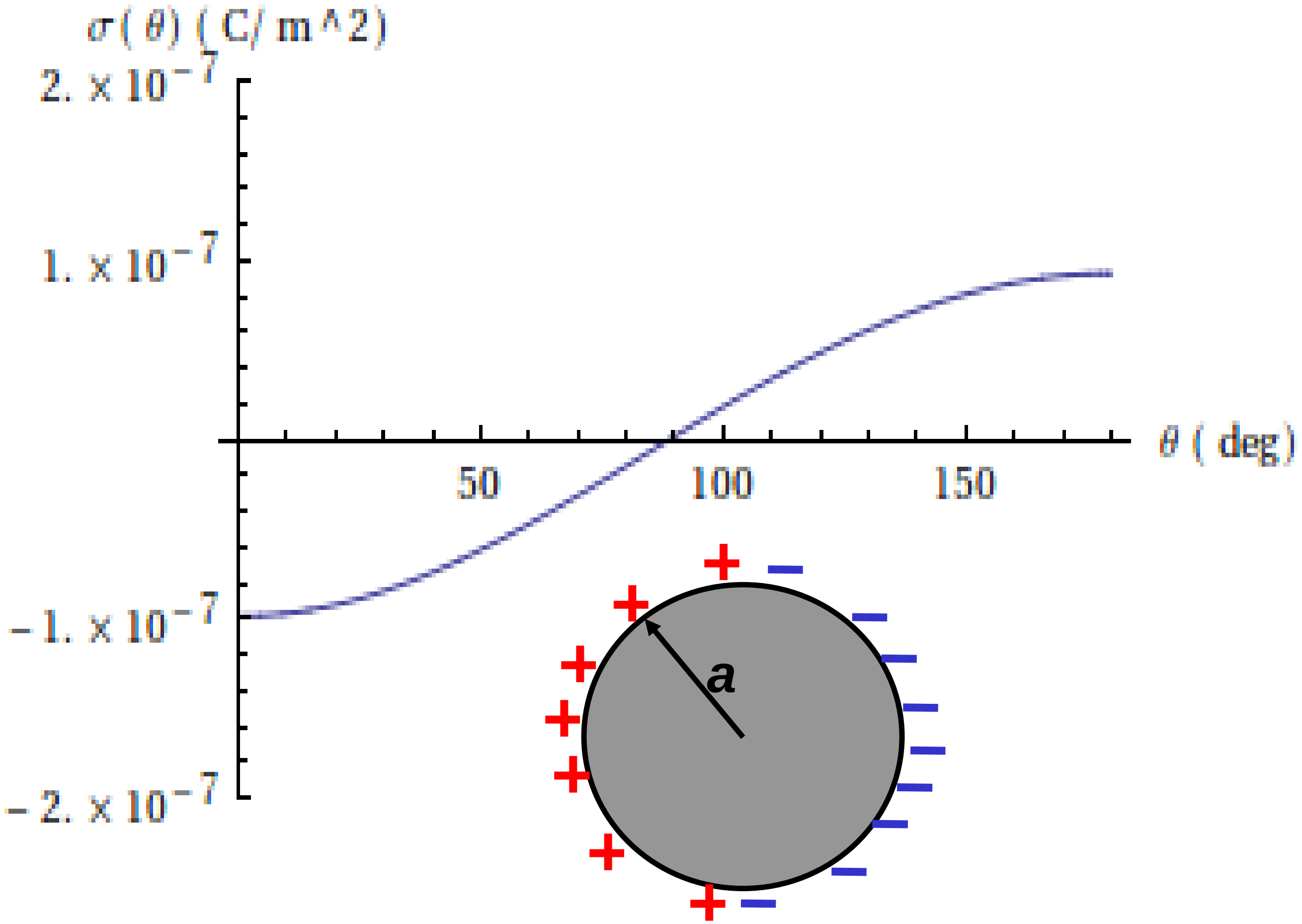
carga puntual + esfera conductora



La esfera queda a potencial CERO. Para ponerla en un potencial arbitrario, puede colocarse una segunda carga imagen en el centro de la esfera conductora.

El método de las imágenes

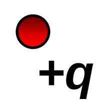
Para esfera descargada



a = 0.1 m

d = 5 m

q = 10^{-5}C



Ecuaciones de Laplace

$$\nabla^2 V = 0$$

Ecuación de Laplace

***La información de V* es considerada pidiendo a la solución general la satisfacción de las condiciones de borde**

Coordenadas rectangulares:

$$\nabla^2 \varphi \equiv \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2}$$

Coordenadas esféricas: sen

$$\nabla^2 \varphi \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi}{\partial r}\right) + \frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial}{\partial \theta}\left(\operatorname{sen}\theta \frac{\partial \varphi}{\partial \theta}\right) + \frac{1}{r^2 \operatorname{sen}^2\theta}\frac{\partial^2 \varphi}{\partial \phi^2}$$

Coordenadas cilíndricas:

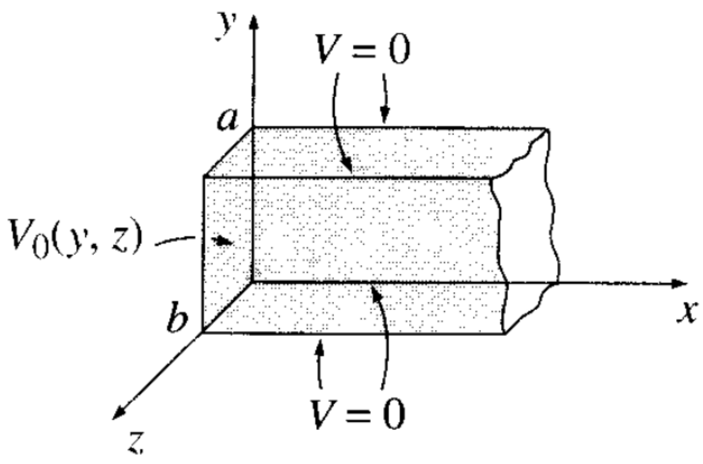
$$\nabla^2 \varphi \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \varphi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2} + \frac{\partial^2 \varphi}{\partial z^2}$$

Separación de Variables

Coordenadas Cartesianas:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0. \quad \longrightarrow \quad V(x, y, z) = X(x)Y(y)Z(z).$$

$$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} + \frac{1}{Z}\frac{d^2 Z}{dz^2} = 0.$$



$$\left.\begin{array}{ll}
\text{(i)} & V = 0 \text{ when } y = 0, \\
\text{(ii)} & V = 0 \text{ when } y = a, \\
\text{(iii)} & V = 0 \text{ when } z = 0, \\
\text{(iv)} & V = 0 \text{ when } z = b, \\
\text{(v)} & V \to 0 \text{ as } x \to \infty, \\
\text{(vi)} & V = V_0(y, z) \text{ when } x = 0.
\end{array}\right\}$$

Separación de Variables

Coordenadas Esféricas:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}=0.$$

Nos vamos a limitar a resolver problemas con **simetría azimutal** (solución independiente de la coordenada ϕ)

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)=0. \longrightarrow V(r,\theta)=R(r)\Theta(\theta).$$

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)=0.$$

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)=l(l+1),\quad \frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)=-l(l+1).$$

Separación de Variables – Coordenadas Esféricas

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = l(l+1)R \longrightarrow R(r) = Ar^l + \frac{B}{r^{l+1}}$$

$$\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) = -l(l+1)\sin\theta\,\Theta \longrightarrow \Theta(\theta) = P_l(\cos\theta).$$

Polinomio de Legendre

$$\boxed{V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta).}$$