TRAZADO DE LINEAS EQUIPOTENCIALES

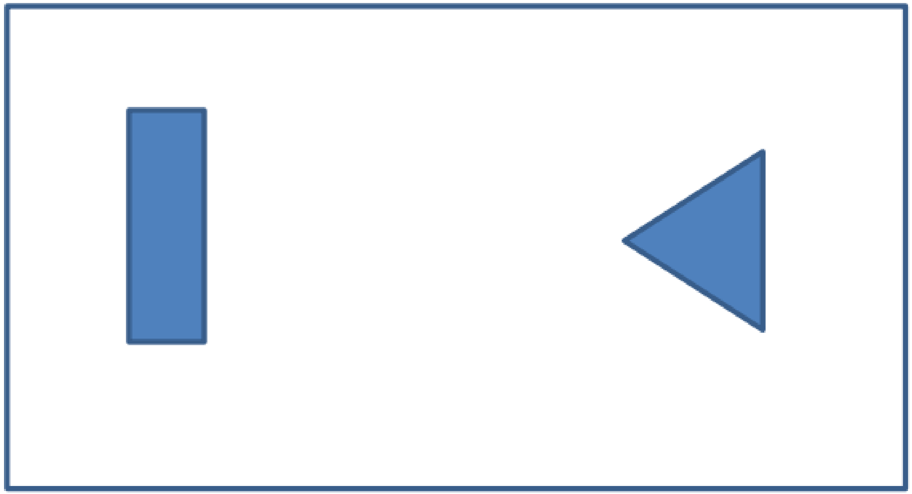
Líneas Equipotenciales: lugar geométrico donde el potencial eléctrico toma el mismo Valor.

Ecuación de Laplace

$$\nabla^2\Phi = 0 \qquad \nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} \qquad \Phi = \Phi(x, y, z)$$

$$\vec{E} = -\textit{Grad}\ \Phi = -\vec{\nabla}\Phi$$

Configuración de conductor rectangular y triangular en equilibrio electrostático, a diferentes potenciales.



Resolución de la Ecuación de Laplace

Métodos analíticos

- *Métodos de la imágenes*
- *Separación de Variables*

Métodos aproximados

- *Método analógicos (experimental).*
- *Métodos Numéricos(método de relajación).*

Método Analógico

Ecuación de Continuidad $$\frac{\partial \rho}{\partial t} + \nabla.\vec{j} = 0$$

Densidad de corriente $$dI = \vec{j}.\breve{n}dA$$

Corrientes estacionarias $$\mathbf{Div}\,\vec{\mathbf{j}} = \nabla.\vec{j} = 0$$

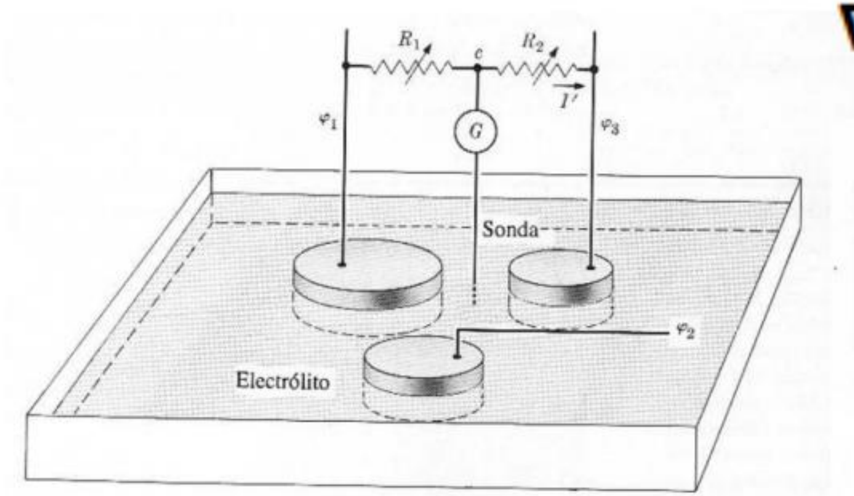
Ley de Ohm $$\vec{j} = \sigma\vec{E}$$ σ =Conductividad

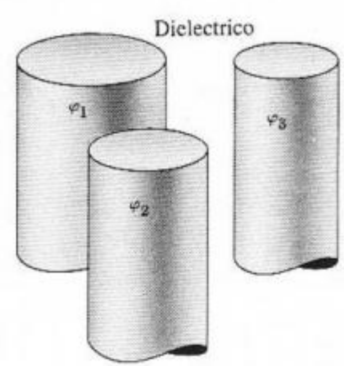
$$\mathbf{Div}\,\vec{\mathbf{j}} = \nabla.\left(\sigma\vec{E}\right) = 0 \Longrightarrow \nabla.\vec{E} = 0$$

$$\nabla \times \vec{E} = 0 \qquad \vec{E} = -\nabla\Phi$$ Campos Electrostáticos

$$\nabla^2\Phi = 0$$

El potencial en el conductor (corrientes Estacionarias) cumple la ecuación de Laplace.





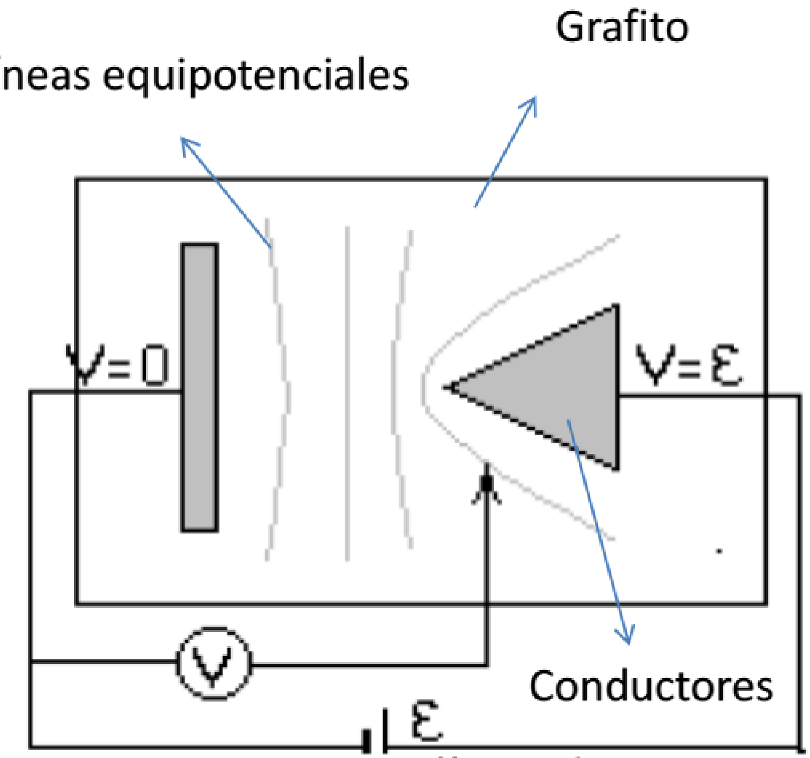
$$\vec{E}_{electrostatico} \Longleftrightarrow \vec{J}_{conductor}$$

$$\Phi_{electrostatico} \Longleftrightarrow \Phi_{conductor}$$

Método Analógico

- *Toman la formas de los Conductores cerca de estos.*
- *Son perpendiculares en los bordes del grafito.*

Líneas equipotenciales



Analogía entre las líneas de campo electrostático *E* en un problema de conductores en un medio dieléctrico y las líneas de densidad de corriente *J;* en un medio conductor óhmico a través del cual se establece una diferencia de potencial.

$$\mathbf{Div}\,\vec{\mathbf{j}} = \nabla . \vec{j} = 0$$

$$\vec{j} = \sigma \vec{E}$$

$$\vec{E} = -\nabla \Phi$$

$$\nabla^2 \Phi = 0$$

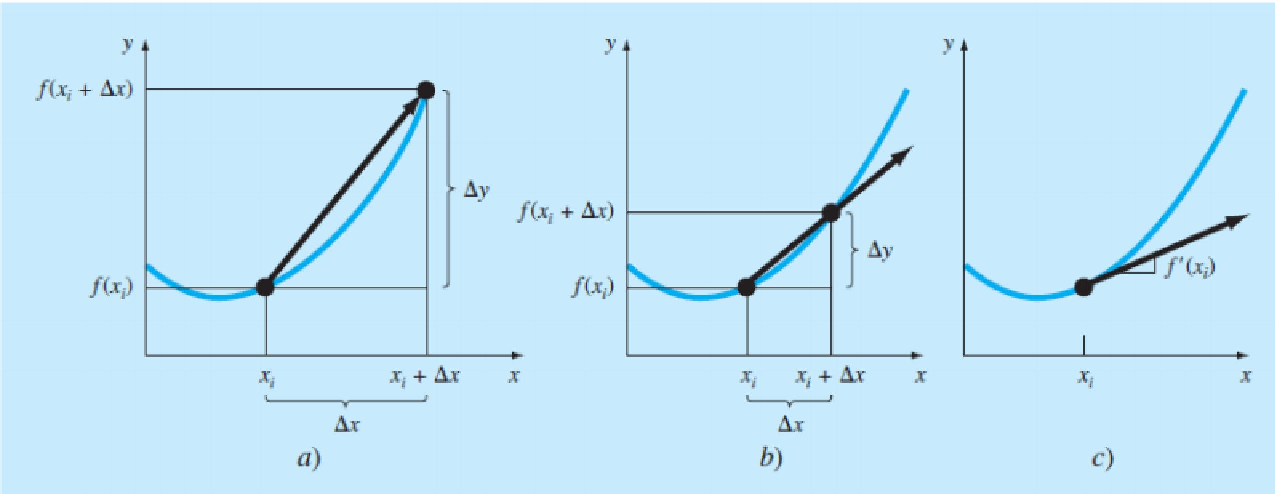
Por medio de mediciones de potencial mediante tester en régimen de corrientes estacionarias resolvemos la ecuación de Laplace.

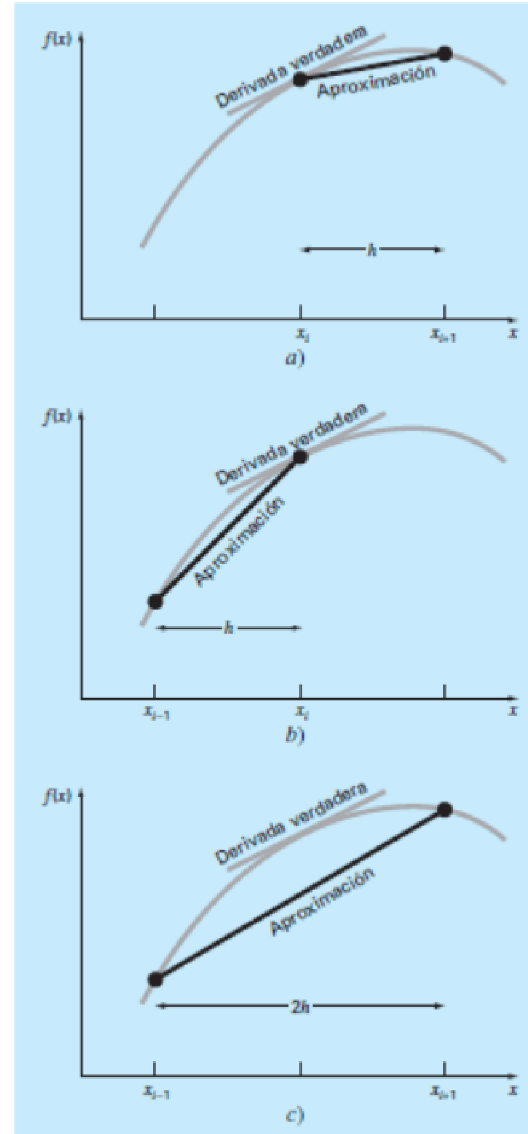
Las *líneas de campo* son perpendiculares a *líneas equipotenciales* (L.E.)

$$\left|\vec{E}\right| \approx \left|\frac{V(x_2) - V(x_1)}{\Delta x}\right|$$

Menor separación de L.E. Implica mayo intensidad del campo eléctrico.

DIFERENCIACION NUMERICA.





$$\frac{\Delta y}{\Delta x} = \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x}$$

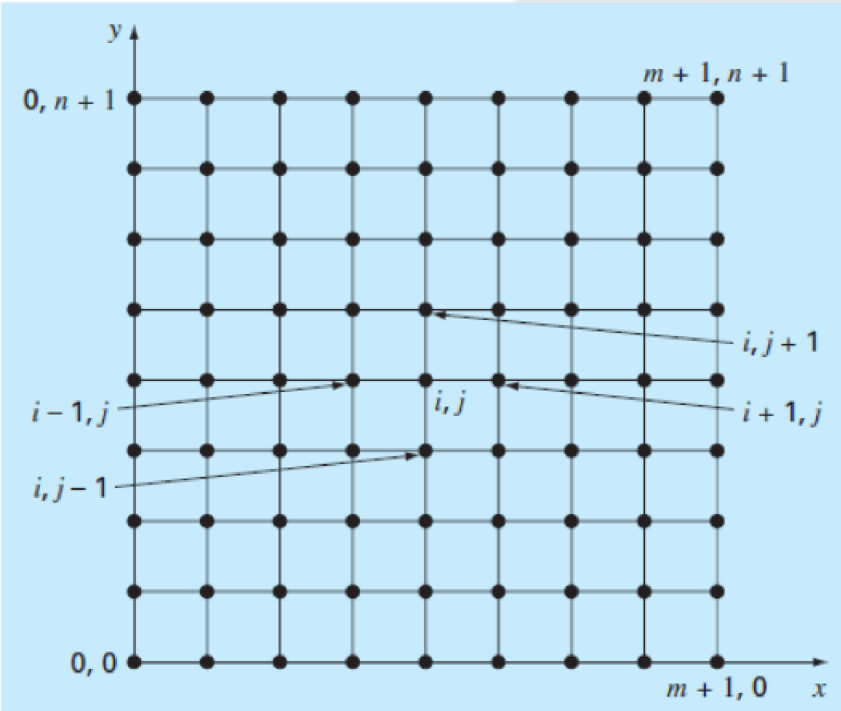
$$f'(x_i) \cong \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

Aproximación primera derivada con diferencias centrales

$$f''(x_i) \cong \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}$$

Aproximación segunda derivada con diferencias centrales

Método de Relajación



Malla para la solución por diferencias finitas de la ecuación de Laplace

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0 \quad \textit{Ec. de Laplace}$$

$$\Delta x=\Delta y=d$$

$$\frac{\partial\Phi}{\partial x}=\frac{\Phi_{i+1,j}-\Phi_{i-1,j}}{2d} \qquad \frac{\partial\Phi}{\partial y}\cong\frac{\Phi_{i,j+1}-\Phi_{i,j-1}}{2d}$$

$$\frac{\partial^2\Phi}{\partial x^2}\cong\frac{\dfrac{\Phi_{i+2,j}-\Phi_{i,j}}{2d}-\dfrac{\Phi_{i,j}-\Phi_{i-2,j}}{2d}}{2d}=\frac{\Phi_{i+2,j}-2\Phi_{i,j}+\Phi_{i-2,j}}{4d^2}$$

$$=\frac{\Phi_{i+1,j}-2\Phi_{i,j}+\Phi_{i-1,j}}{d^2}$$

$$\frac{\partial^2\Phi}{\partial x^2}\cong\frac{\Phi_{i+1,j}-2\Phi_{i,j}+\Phi_{i-1,j}}{d^2} \qquad \frac{\partial^2\Phi}{\partial y^2}\cong\frac{\Phi_{i,j+1}-2\Phi_{i,j}+\Phi_{i,j-1}}{d^2}$$

$$\frac{\Phi_{i+1,j}-2\Phi_{i,j}+\Phi_{i-1,j}}{d^2}+\frac{\Phi_{i,j+1}-2\Phi_{i,j}+\Phi_{i,j-1}}{d^2}=0 \Rightarrow \Phi_{i+1,j}+\Phi_{i-1,j}+\Phi_{i,j+1}-4\Phi_{i,j}+\Phi_{i,j-1}=0$$

Se resuelve en forma Iterativa-> ***Met. Relajación***

$$\boxed{\Phi_{i,j}=\frac{\Phi_{i+1,j}+\Phi_{i-1,j}+\Phi_{i,j+1}+\Phi_{i,j-1}}{4}}$$

$\Phi_{i,j}$ que satisface la ecuación de Laplace

Método de Relajación

Condiciones de Borde

Condiciones de borde de Dirichlet

$$\mathbf{\Phi_{contorno}(conductor) = \Phi_{contorno}(grafito)}$$

Condiciones de borde de Neumann

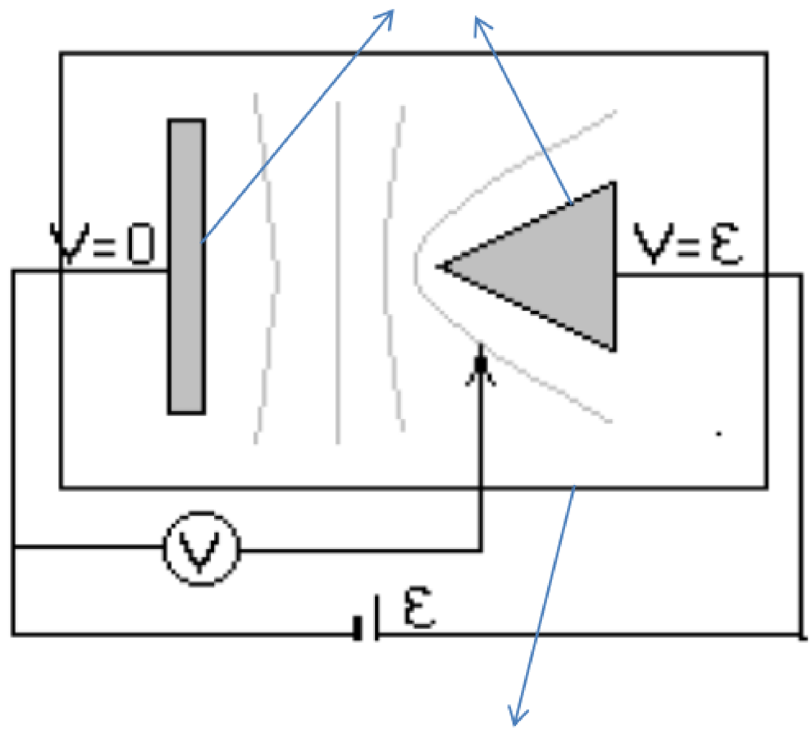
$$J_n\big|_{borde} = 0$$

$$\vec{J} = \sigma \cdot \vec{E}$$

$$J_n\big|_{borde} = \sigma \cdot E_n\big|_{borde} = 0 \quad \Rightarrow \quad E_n\big|_{borde} = 0$$

$$E_t\big|_{borde} \neq 0$$

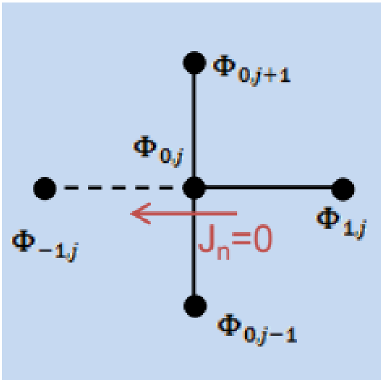
Contorno de conductor-> Dirichelt



Contorno de grafito=> Neumann
(condición de continuidad entre
Conductor y dieléctrico)

Condiciones de borde de Neumann

En extremos de la placa:



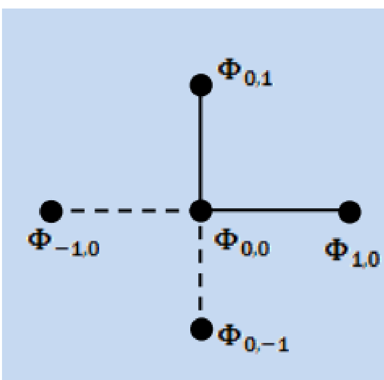
$$\Phi_{1,j} + \Phi_{-1,j} + \Phi_{0,j+1} + \Phi_{0,j-1} - 4\Phi_{0,j} = 0$$

$$\frac{\partial \Phi}{\partial x} \cong \frac{\Phi_{1,j} - \Phi_{-1,j}}{2d} \qquad \Phi_{-1,j} \cong \Phi_{1,j} - 2d\frac{\partial \Phi}{\partial x}$$

$$2\Phi_{1,j} - 2d\frac{\partial \Phi}{\partial x} + \Phi_{0,j+1} + \Phi_{0,j-1} - 4\Phi_{0,j} = 0$$

$E_n=0$ $$\frac{\partial \Phi}{\partial x} = 0 \Rightarrow \boxed{\Phi_{0,j} = \frac{2\Phi_{1,j} + \Phi_{0,j+1} + \Phi_{0,j-1}}{4}}$$ C.B. Extremos

En esquinas de la placa: (i=0, j=0)



$$2\Phi_{1,0} - 2d\frac{\partial \Phi}{\partial x} + \Phi_{0,1} + \Phi_{0,-1} - 4\Phi_{0,0} = 0$$

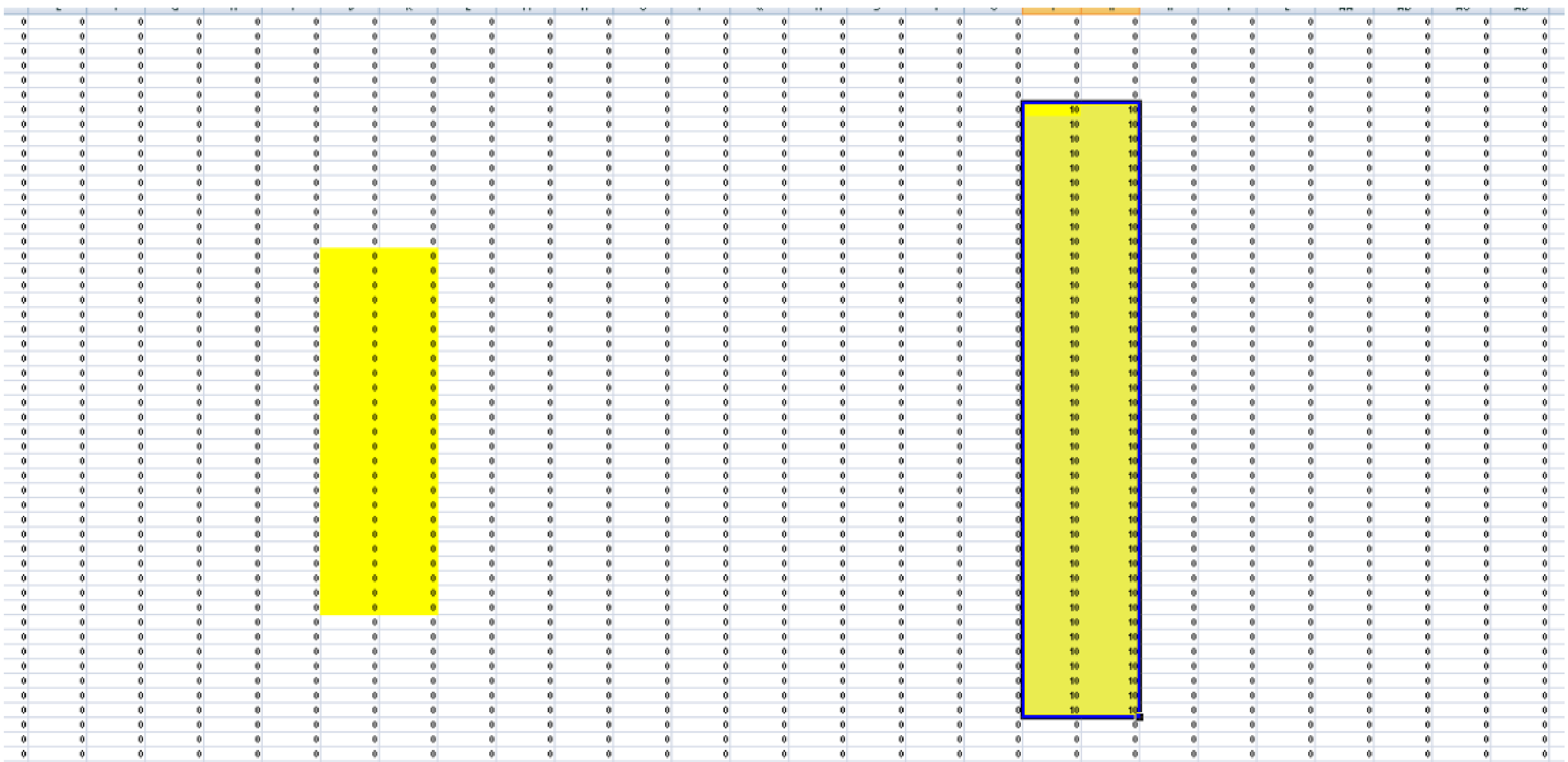
$$\frac{\partial \Phi}{\partial y} \cong \frac{\Phi_{1,0} - \Phi_{-1,0}}{2d} \qquad \Phi_{0,-1} \cong \Phi_{0,1} - 2d\frac{\partial \Phi}{\partial y}$$

$$2\Phi_{1,0} - 2d\frac{\partial \Phi}{\partial x} + 2\Phi_{0,1} - 2d\frac{\partial \Phi}{\partial y} - 4\Phi_{0,0} = 0$$

$E_n=0$ $$\frac{\partial \Phi}{\partial x} = 0 \qquad \frac{\partial \Phi}{\partial y} = 0 \Rightarrow 4\Phi_{0,0} = 2\Phi_{1,0} + 2\Phi_{0,1} \Rightarrow \boxed{\Phi_{0,0} = \frac{\Phi_{1,0} + \Phi_{0,1}}{2}}$$ C.B. Esquinas

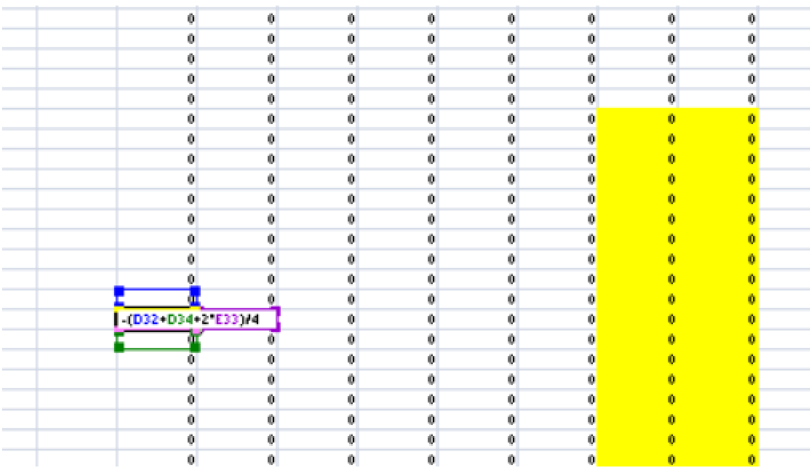
Implementación del Método de Relajación en Excel

En una hoja de cálculo (tipo Excel), se establecerá el valor de potencial de cada celda como el valor promedio de los cuatro vecinos más próximos. Las regiones conductoras se definirán imponiendo a cada celda el mismo valor de potencial que su vecina. En las celdas que constituyen los límites de la hoja de cálculo se impondrán las condiciones de borde adecuadas.

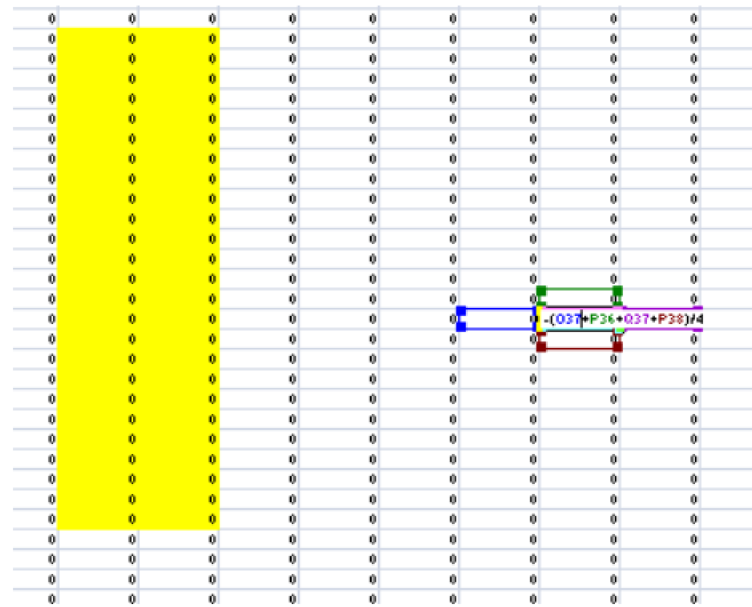


Implementación del Método de Relajación en Excel

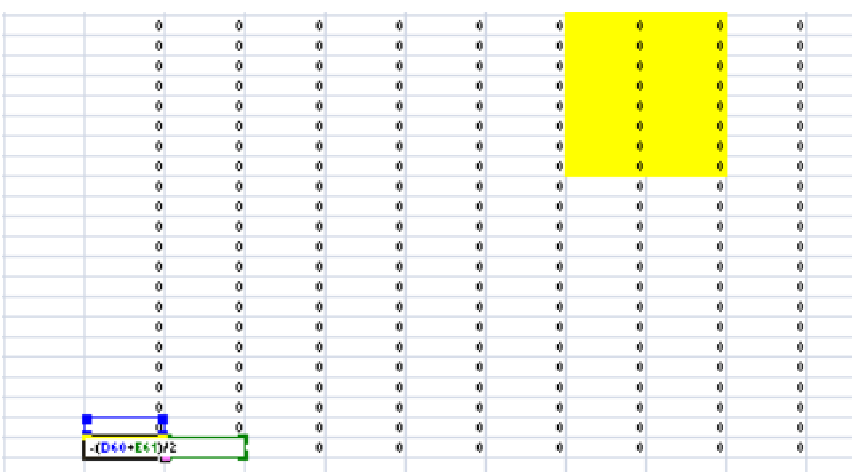
Contorno placa grafito



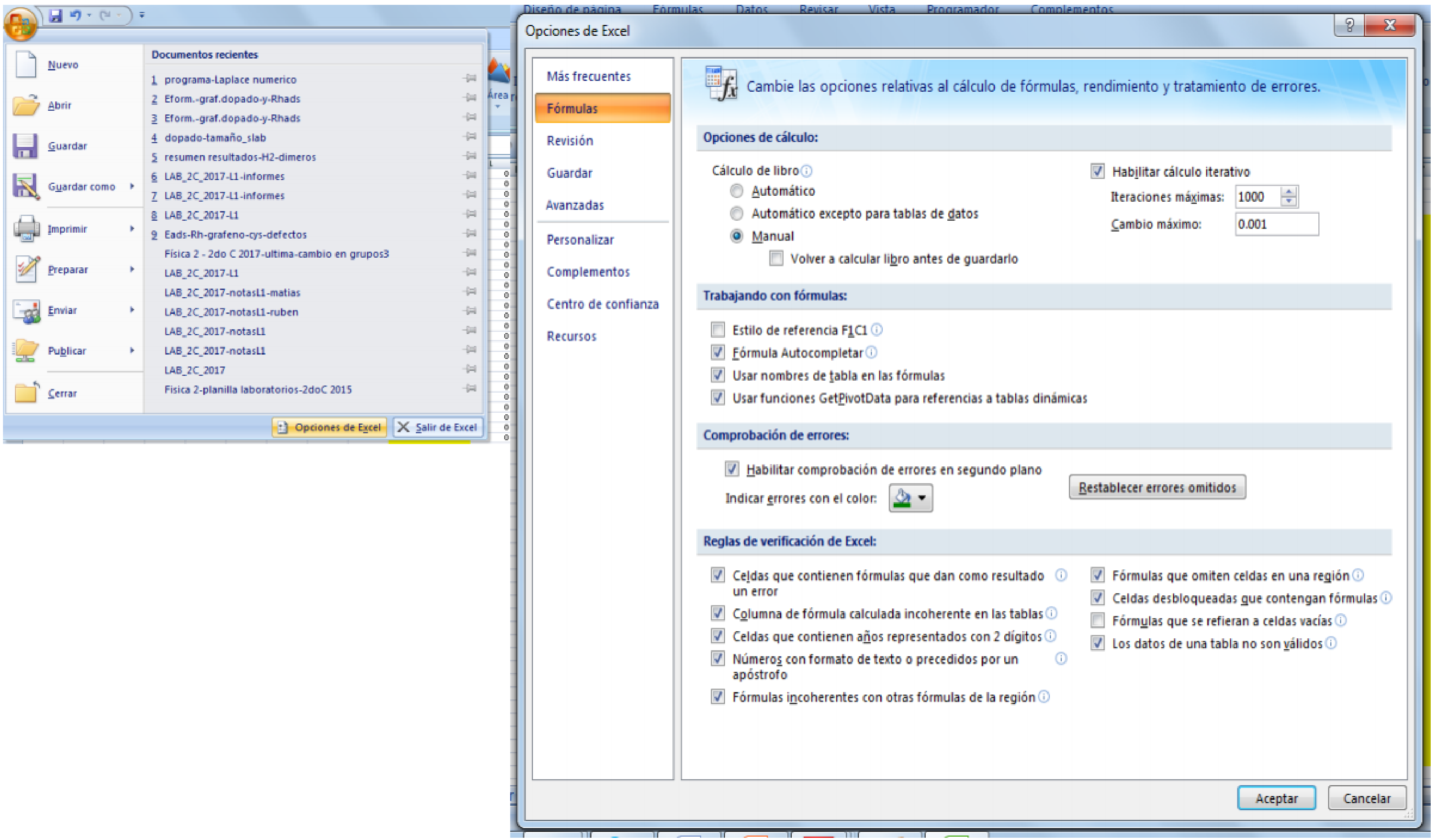
Ecuación de Laplace numerica



Esquinas placa grafito



Implementación del Método de Relajación en Excel



Implementación del Método de Relajación en Excel

Presionar F9 para lanzar el calculo.

D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD
5.4944936	5.5086748	5.5501875	5.6187355	5.7138045	5.8346285	5.9801479	6.1489673	6.3393143	6.5490034	6.7754007	7.0153836	7.2652837	7.5208004	7.7768783	8.0275625	8.2659197	8.484257	8.6750884	8.8332748	8.9584994	9.0543307	9.1256042	9.1767253	9.2110547	9.2308546	9.2374076
5.4074763	5.4220049	5.4646063	5.5350486	5.6329162	5.7575643	5.9080652	6.0831554	6.2811899	6.5001077	6.7374039	6.990098	7.2546744	7.5269622	7.8019075	8.0731927	8.3327148	8.5701976	8.7739316	8.9347617	9.0538505	9.1394064	9.1999992	9.2420478	9.2697298	9.2855321	9.2907509
5.294095	5.3090349	5.3529329	5.4256523	5.5269218	5.6562717	5.8129598	5.9959008	6.2036122	6.4341054	6.6852776	6.9541105	7.2374426	7.5314611	7.8314967	8.1313926	8.4222679	8.6905276	8.9162574	9.0785272	9.1832484	9.249947	9.2934339	9.322229	9.3407748	9.3512824	9.354765
5.1534328	5.168815	5.214123	5.2893583	5.3944583	5.5292032	5.6931069	5.8853166	6.1045424	6.349036	6.6166189	6.904743	7.2105498	7.5308705	7.8620531	8.1993391	8.5350644	8.8539268	9.1225149	9.2802787	9.3510971	9.384129	9.4019942	9.4130976	9.4202985	9.4244993	9.4259528
4.9844958	5.0003032	5.0469967	5.1247786	5.2338888	5.3744662	5.5463826	5.7490874	5.9815054	6.242021	6.5285592	6.8387434	7.1700982	7.5202716	7.8872528	8.2694817	8.6652453	9.0680125	9.4399237	9.5692824	9.55706	9.5338279	9.5176843	9.5082488	9.5032102	9.5008545	9.5002305
4.7863098	4.8024555	4.85031	4.9303671	5.0433095	5.1898003	5.3702249	5.5844384	5.8315952	6.1101318	6.4179195	6.7525499	7.111708	7.4936412	7.8978682	8.3266304	8.7888282	9.3132112	10	10	9.7742552	9.6767105	9.6269695	9.599326	9.5837736	9.5758181	9.5734184
4.5580612	4.5743573	4.6228567	4.7044762	4.8205493	4.972522	5.1615468	5.3880543	5.6514466	5.9500594	6.2814243	6.6427282	7.0313466	7.4454163	7.8845323	8.3507971	8.8505252	9.3960986	10	10	9.863405	9.771999	9.7144034	9.6785822	9.6570248	9.6455171	9.6419411
4.2993013	4.3154143	4.3636204	4.4454392	4.5631596	4.7194172	4.9165604	5.1559021	5.4371306	5.7582174	6.1150955	6.5064128	6.9262621	7.3727711	7.844561	8.3418871	8.8666196	9.4207348	10	10	9.9074786	9.83364	9.7802619	9.7437986	9.7204656	9.7075304	9.7034253
4.0102383	4.0256307	4.0720031	4.1517032	4.2683988	4.4265502	4.6304496	4.882883	5.1839161	5.5306776	5.918349	6.3415068	6.7951719	7.2754027	7.7795051	8.305905	8.8535363	9.4202865	10	10	9.9329549	9.8749491	9.829367	9.7960694	9.7737078	9.7609199	9.7567938
3.6921471	3.7060087	3.7481787	3.8220633	3.9332385	4.0889499	4.2967732	4.5621816	4.8858354	5.2630298	5.6860515	6.1467564	6.6380983	7.1546565	7.6925482	8.2489821	8.8215108	9.4069313	10	10	9.9494588	9.9039368	9.8663186	9.8375559	9.8175417	9.8058192	9.8019886
3.3479179	3.3591045	3.3936448	3.4561091	3.5544826	3.7001379	3.9063679	4.1840447	4.5349747	4.9502621	5.41672	5.9219537	6.456321	7.0130104	7.5873958	8.1762173	8.7767461	9.3859766	10	10	9.9609966	9.9251032	9.8945214	9.8704186	9.8532205	9.8429688	9.8395873
2.9827252	2.9897556	3.0120741	3.0551023	3.1292652	3.2515308	3.4452538	3.7333509	4.1204116	4.5869349	5.1091752	5.6685248	6.2526675	6.8540451	7.4681083	8.0919642	8.7234116	9.3602709	10	10	9.969467	9.941025	9.9163322	9.896479	9.8820656	9.8733655	9.8704765
2.6047058	2.60591	2.6105613	2.6236956	2.65664	2.7321183	2.8903772	3.1842698	3.6269309	4.168404	4.7649969	5.3907354	6.0321605	6.6827173	7.3392865	8.0003073	8.6647785	9.3317315	10	10	9.975881	9.953252	9.9333747	9.9171836	9.9052899	9.8980479	9.8956319
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1.858962	1.845275	1.8024507	1.7258957	1.6084485	1.4476325	1.2939876	1.5280883	2.3316244	3.1854521	4.0107862	4.8051283	5.5762041	6.3310355	7.0748171	7.8112728	8.5430747	9.2721625	10	10	9.9847031	9.9702493	9.957399	9.9467839	9.9388833	9.9340246	9.9323949
1.5207026	1.5008584	1.4378474	1.3199716	1.118525	0.7578638	0	0	1.5780891	2.7044621	3.6607397	4.5361362	5.3660242	6.1676566	6.9508789	7.721951	8.4851707	9.2436896	10	10	9.9877488	9.976153	9.9658127	9.9572402	9.9508377	9.9468898	9.9455637
1.2227391	1.1999834	1.1284482	0.997901	0.7880133	0.465358	0	0	1.2764	2.3937335	3.3917558	4.3128345	5.1842701	6.0228382	6.8392142	7.6405723	8.4320229	9.2174424	10	10	9.9901545	9.9808257	9.9724908	9.9655647	9.9603801	9.9571775	9.9561008
0.9707775	0.9481875	0.8783264	0.7553842	0.5704087	0.3155976	0	0	1.1338648	2.2024355	3.1998517	4.1393161	5.0355177	5.9003316	6.7426666	7.5691748	8.3849508	9.1940711	10	10	9.9920565	9.9845246	9.9777868	9.9721793	9.9679755	9.9653757	9.9645013
0.7643886	0.7439009	0.6814936	0.5750637	0.4227434	0.2266559	0	0	1.0566843	2.082379	3.0660015	4.0091683	4.9182577	5.8003989	6.6620248	7.5085684	8.3445706	9.1739026	10	10	9.9935574	9.9874459	9.9819744	9.9774163	9.9739959	9.9718789	9.9711666
0.5992879	0.5817225	0.5288468	0.4407599	0.3189245	0.1683072	0	0	1.0105372	2.0044584	2.9726839	3.9131804	4.8280273	5.7210558	6.5965279	7.4585502	8.3108894	9.1569776	10	10	9.994736	9.9897411	9.9852669	9.9815372	9.9787366	9.9770025	9.9764189
0.4695669	0.4550042	0.4115398	0.3403033	0.243949	0.1276674	0	0	0.9810381	1.9522812	2.9071537	3.8429053	4.7596783	5.6593275	6.5445302	7.4182525	8.2834819	9.1431256	10	10	9.9956527	9.9915269	9.9878299	9.9847469	9.9824309	9.9809964	9.9805135
0.3691694	0.3573065	0.322107	0.2650425	0.1889489	0.0984285	0	0	0.9613579	1.9165103	2.8607885	3.7916577	4.7085022	5.612091	6.5040515	7.3864769	8.2616782	9.1320484	10	10	9.996354	9.9928934	9.9897918	9.9872045	9.9852604	9.984056	9.9836505
0.2926559	0.2830405	0.2546203	0.2088727	0.1484139	0.0771096	0	0	0.9479012	1.8916408	2.8278665	3.7544723	4.6706197	5.5765186	6.4731386	7.3619485	8.24472	9.1233944	10	10	9.9968751	9.9939088	9.9912499	9.9890315	9.9873643	9.9863312	9.9859834
0.2355008	0.2276556	0.2045262	0.1674638	0.1187551	0.0616057	0	0	0.9386199	1.8743062	2.8045906	3.7277748	4.6430157	5.5502531	6.4500597	7.343477	8.2310701	9.1160126	10	10	9.997242	9.9946238	9.9922767	9.9903182	9.9888461	9.9879338	9.9876265
0.1941397	0.1876172	0.1684185	0.1377414	0.0975621	0.0505662	0	0	0.9322831	1.8623898	2.7884356	3.7090436	4.6234389	5.5314407	6.4333893	7.3300444	8.22248	9.1119888	10	10	9.9974728	9.9950736	9.9929227	9.9911277	9.9897784	9.9889421	9.9886603
0.1659092	0.1603069	0.1438331	0.117555	0.0832066	0.0431035	0	0	0.9281313	1.8545475	2.7777351	3.6965439	4.6102747	5.5186996	6.4220282	7.3208434	8.2160241	9.1086648	10	10	9.997579	9.9952805	9.9932197	9.9914998	9.9902069	9.9894054	9.9891353
0.1489559	0.1439123	0.1290898	0.1054674	0.0746238	0.0386468	0	0	0.9257015	1.8499445	2.7714274	3.6891375	4.6024325	5.51107	6.4151934	7.3152871	8.2121145	9.1066484	10	10	9.9975656	9.9952544	9.9931821	9.9914525	9.9901522	9.989346	9.9890742
0.1421536	0.1373355	0.1231795	0.1006266	0.07119	0.0368651	0	0	0.9247364	1.8481111	2.7689043	3.6861597	4.5992616	5.5079677	6.4123998	7.313006	8.210504	9.105816	10	10	9.997432	9.9949937	9.9928075	9.9909829	9.989611	9.9887605	9.9884735
0.1450457	0.1401327	0.125697	0.1026931	0.072659	0.0376285	0	0	0.9251385	1.8488677	2.7699301	3.6873476	4.6004994	5.5091515	6.4134427	7.3138412	8.2110847	9.1061132	10	10	9.9971712	9.9944853	9.9920771	9.9900672	9.9885561	9.9876191	9.9873029
0.1578202	0.1524875	0.1368126	0.1118126	0.079139	0.0409949	0	0	0.9269552	1.8522993	2.7746117	3.6928131	4.6062493	5.5147081	6.4183886	7.3178393	8.2138856	9.1075537	10	10	9.9967701	9.9937035	9.9909541	9.9886595	9.9869345	9.9858649	9.9855038
0.1813174	0.1752205	0.1572842	0.1286296	0.0911042	0.0472172	0	0	0.9303885	1.8587714	2.7834152	3.7030567	4.6169896	5.5250557	6.4275749	7.3252504	8.2190699	9.1102178	10	10	9.9962085	9.9926088	9.9893819	9.9866893	9.9846653	9.9834104	9.9829865
0.2170694	0.209832	0.1885076	0.1543429	0.1094472	0.0567754	0	0	0.9358334	1.8689923	2.7972335	3.7190229	4.6336113	5.5409637	6.4416171	7.3365268	8.2269318	9.1142497	10	10	9.9954579	9.9911462	9.9872817	9.984058	9.9816354	9.9801335	9.9796261
0.2673647	0.2585742	0.2326089	0.1908164	0.1355846	0.0704437	0	0	0.9439602	1.8841424	2.8175182	3.7422064	4.657486	5.5635868	6.4614168	7.3523187	8.2378877	9.1198515	10	10	9.9944803	9.9892414	9.9845476	9.9806338	9.9776938	9.9758717	9.9752562
0.3353204	0.3245424	0.2925811	0.2407633	0.1716526	0.0894228	0	0	0.9558737	1.9061127	2.8465081	3.7748186	4.69056	5.5944998	6.4881613	7.3734565	8.2524569	9.1272718	10	10	9.9932253	9.9867973	9.9810413	9.976245	9.9726445	9.9704143	9.9696609
0.4249262	0.4117548	0.3724626	0.3080439	0.2208659	0.1156042	0	0	0.973433	1.9379442	2.8876053	3.8200244	4.7354608	5.6357149	6.5232924	7.4009047	8.2712215	9.1367822	10	10	9.9916278	9.9836878	9.976584	9.970671	9.9662368	9.9634923	9.9625655
0.5409888	0.5251617	0.4775345	0.3981335	0.2881949	0.1521397	0	0	0.9999287	1.9846483	2.9459726	3.8822441	4.7955754	5.6896359	6.5684139	7.4356679	8.2947543	9.14864	10	10	9.9896029	9.9797499	9.9709463	9.9636304	9.958153	9.9547671	9.9536243
0.6888444	0.6704591	0.6144589	0.5188224	0.3816806	0.2047749	0	0	1.0416526	2.0547774	3.0294295	3.967444	4.8750008	5.7588761	6.6250906	7.4786224	8.3235029	9.1630289	10	10	9.9870394	9.9747715	9.963833	9.9547658	9.9479938	9.9438155	9.9424067
0.8736409	0.8534823	0.7911171	0.6810943	0.5149813	0.2852989	0	0	1.1119306	2.1634191	3.1495718	4.0831528	4.9781581	5.845823	6.6944891	7.5302574	8.3576247	9.1799794	10	10	9.9837899	9.9684744	9.9548625	9.9436229	9.9352597	9.9301145	9.9283822
1.0989627	1.078849	1.0155542	0.899555	0.7119184	0.4214662	0	0	1.242688	2.3374511	3.3223489	4.2375033	5.1087192	5.9518262	6.7768332	7.5903296	8.3967818	9.1992721	10	10	9.9796538	9.9604863	9.9435366	9.9296239	9.9193298	9.9130242	9.9109059
1.3647658	1.3475645	1.2920472	1.1897793	1.0117615	0.6086877	0	0	1.5214271	2.6214241	3.5649532	4.4350769	5.2674691	6.0760004	6.8707468	7.65749	8.4399281	9.2203372	10	10	9.9743483	9.9502956	9.9291937	9.9120303	9.8994383	9.8917743	9.8892085
1.6652786	1.6539997	1.6186787	1.5551167	1.4567875	1.3215932	1.1967049	1.4352721	2.2216923	3.0619718	3.8802727	4.6736887	5.449379	6.2140462	6.9727349	7.7290088	8.4851365	9.2421604	10	10	9.967455	9.9371725	9.9109364	9.889894	9.8746505	9.8654596	9.8623978
1.9887155	1.984721	1.9729813	1.9554306	1.9388607	1.9443449	2.0300911	2.322829	2.8682405	3.5246415	4.2206177	4.9293587	5.6424326	6.3581751	7.0772235	7.8007374	8.5294881	9.263182	10	10	9.958313	9.9200248	9.8875142	9.8619931	9.8438482	9.8330552	9.8294853
2.3205708	2.3234749	2.33337	2.3550216	2.3991175	2.4870514	2.6566875	2.9579064	3.4039878	3.9479182	4.5483709	5.1808562	5.8329617	6.4991226	7.1773476	7.8673043	8.5689433	9.281096	10	10	9.9457888	9.897126	9.8571375	9.826757	9.805739	9.7934751	9.7894588
2.6471124	2.6555692	2.682323	2.7324745	2.8158256	2.9483281	3.151958	3.4483657	3.8421192	4.3148933	4.844298	5.4129231	6.0096044	6.6281514	7.2658583	7.9222772	8.5979396	9.2922784	10	10	9.9277366	9.8655856	9.8171951	9.7822075	9.7589298	9.7457033	9.7414302
2.9572991	2.9697409	3.008243	3.0770799	3.1837194	3.3387985	3.5547559	3.841769	4.2015045	4.6254959	5.1012442	5.6171524	6.1645761	6.738188	7.3357941	7.9581093	8.6083243	9.2901015	10	10	9.899598	9.8203248	9.7639013	9.7260073	9.7021332	9.6890442	9.6848917
3.2432226	3.2582678	3.3042348	3.3842776	3.5035542	3.6687553	3.8868461	4.1627806	4.4969465	4.8846344	5.318302	5.7901138	6.2935805	6.8244212	7.3811784	7.9661614	8.5872238	9.2598323	10	10	9.8503642	9.7522652	9.6921406	9.6558582	9.634627	9.6235265	9.6200902
3.4997339	3.5163265	3.5665956	3.652675	3.7778839	3.9462259	4.1614789	4.4259282	4.7392137	5.0981184	5.4975171	5.9316955	6.3954578	6.884953	7.3985167	7.938274	8.5146713	9.1620425	10	10	9.7496408	9.6462969	9.5966147	9.570742	9.5570789	9.5504351	9.5484649
3.7237905	3.7411965	3.7936256	3.8824107	4.0095348	4.1772235	4.3873351	4.6406393	4.93624	5.2714632	5.642282	6.0439949	6.4718741	6.9216558	7.3898654	7.8739112	8.371264	8.8737263	9.3354599	9.5019703	9.5019757	9.4867524	9.4773734	9.473516	9.4726143	9.4727742	9.4729548
3.9138118	3.931561	3.984809	4.0743054	4.2011048	4.3662655	4.5704473	4.8134819	5.0940488	5.4095933	5.7565069	6.130454	6.5266836	6.9401943	7.3656072	7.7964344	8.2229031	8.6262587	8.9662439	9.1705448	9.2696423	9.321472	9.3527226	9.3734496	9.3872051	9.3952103	9.397868
4.0691502	4.0869699	4.1402783	4.2294202	4.3548223	4.5167781	4.7151795	4.9492435	5.2173083	5.5167569	5.8440741	6.1949778	6.5645289	6.9471159	7.3361876	7.7235366	8.0978438	8.4423223	8.7320545	8.9444557	9.0847066	9.1769002	9.2307253	9.2804052	9.3076769	9.3231245	9.328165
4.1896966	4.2074528	4.2604685	4.3488178	4.4725143	4.6313567	4.8247409	5.0514744	5.3096304	5.5964726	5.9084482	6.2412189	6.5896729	6.947857	7.3087639	7.6639237	8.0028278	8.3125225	8.5785678	8.7898765	8.9479881	9.0628444	9.1449384	9.2022328	9.2400356	9.2615879	9.2686163

Implementación del Método de Relajación en Excel