En las siguientes ecuaciones ϕ y ψ representan escalares, $\boldsymbol{a}$, $\boldsymbol{b}$ y $\boldsymbol{c}$ son vectores.

Triple productos y productos mixtos

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b})$$

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) + \boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{a}) + \boldsymbol{c} \times (\boldsymbol{a} \times \boldsymbol{b}) = 0$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b} \cdot (\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{c} \cdot (\boldsymbol{a} \times \boldsymbol{b}) \ \ (\equiv [\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}])$$

$$(\boldsymbol{a} \times \boldsymbol{b}) \cdot (\boldsymbol{c} \times \boldsymbol{d}) = (\boldsymbol{a} \cdot \boldsymbol{c})(\boldsymbol{b} \cdot \boldsymbol{d}) - (\boldsymbol{a} \cdot \boldsymbol{d})(\boldsymbol{b} \cdot \boldsymbol{c})$$

Reglas de producto

$$\boldsymbol{\nabla}(\phi\psi) = \phi\boldsymbol{\nabla}\psi + \psi\boldsymbol{\nabla}\phi$$

$$\boldsymbol{\nabla}(\boldsymbol{a} \cdot \boldsymbol{b}) = \boldsymbol{a} \times (\boldsymbol{\nabla} \times \boldsymbol{b}) + \boldsymbol{b} \times (\boldsymbol{\nabla} \times \boldsymbol{a}) + (\boldsymbol{a} \cdot \boldsymbol{\nabla})\boldsymbol{b} + (\boldsymbol{b} \cdot \boldsymbol{\nabla})\boldsymbol{a}$$

$$\boldsymbol{\nabla} \cdot (\phi\boldsymbol{a}) = \phi\boldsymbol{\nabla} \cdot \boldsymbol{a} + \boldsymbol{a} \cdot \boldsymbol{\nabla}\phi$$

$$\boldsymbol{\nabla} \cdot (\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{b} \cdot (\boldsymbol{\nabla} \times \boldsymbol{a}) - \boldsymbol{a} \cdot (\boldsymbol{\nabla} \times \boldsymbol{b})$$

$$\boldsymbol{\nabla} \times (\phi\boldsymbol{a}) = \phi\boldsymbol{\nabla} \times \boldsymbol{a} - \boldsymbol{a} \times \boldsymbol{\nabla}\phi$$

$$\boldsymbol{\nabla} \times (\boldsymbol{a} \times \boldsymbol{b}) = (\boldsymbol{b} \cdot \boldsymbol{\nabla})\boldsymbol{a} - (\boldsymbol{a} \cdot \boldsymbol{\nabla})\boldsymbol{b} + \boldsymbol{a}\boldsymbol{\nabla} \cdot \boldsymbol{b} - \boldsymbol{b}\boldsymbol{\nabla} \cdot \boldsymbol{a}$$

$$\boldsymbol{\nabla} \times (\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{a} \times (\boldsymbol{\nabla} \times \boldsymbol{b}) - \boldsymbol{b} \times (\boldsymbol{\nabla} \times \boldsymbol{a}) - (\boldsymbol{a} \times \boldsymbol{\nabla}) \times \boldsymbol{b} + (\boldsymbol{b} \times \boldsymbol{\nabla}) \times \boldsymbol{a}$$

Derivadas segundas

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{a}) = 0$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla}\phi) = 0$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{a}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{a}) - \nabla^2\boldsymbol{a}$$

Teoremas integrales fundamentales

$$\int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} (\boldsymbol{\nabla}\phi) \cdot d\boldsymbol{l} = \phi(\boldsymbol{r}_2) - \phi(\boldsymbol{r}_1)$$

$$\int_V (\boldsymbol{\nabla} \cdot \boldsymbol{a}) d^3r = \int_{S(V)} \boldsymbol{a} \cdot d\boldsymbol{s}$$

$$\int_{S(C)} (\boldsymbol{\nabla} \times \boldsymbol{a}) \cdot d\boldsymbol{s} = \oint_C \boldsymbol{a} \cdot d\boldsymbol{l}$$

Otros teoremas integrales

$$\int_V (\boldsymbol{\nabla}\phi) \ d^3r = \int_{S(V)} d\boldsymbol{s} \ \phi$$

$$\int_V (\boldsymbol{\nabla} \times \boldsymbol{a}) \ d^3r = \int_{S(V)} d\boldsymbol{s} \times \boldsymbol{a}$$

Operadores vectoriales

Coordenadas cartesianas

$d\boldsymbol{l} = dx\ \hat{\boldsymbol{x}} + dy\ \hat{\boldsymbol{y}} + dz\ \hat{\boldsymbol{z}}$

$d^3r = dx\ dy\ dz$

$\boldsymbol{\nabla}\phi = \partial_x\phi\ \hat{\boldsymbol{x}} + \partial_y\phi\ \hat{\boldsymbol{y}} + \partial_z\phi\ \hat{\boldsymbol{z}}$

$\boldsymbol{\nabla}\cdot\boldsymbol{a} = \partial_x a_x + \partial_y a_y + \partial_z a_z$

$\boldsymbol{\nabla}\times\boldsymbol{a} = (\partial_y a_z - \partial_z a_y)\ \hat{\boldsymbol{x}} + (\partial_z a_x - \partial_x a_z)\ \hat{\boldsymbol{y}} + (\partial_x a_y - \partial_y a_x)\ \hat{\boldsymbol{z}}$

$\nabla^2\phi = {\partial_x}^2\phi + {\partial_y}^2\phi + {\partial_z}^2\phi$

$d\boldsymbol{S} = dy\ dz\ \hat{x} + dx\ dzy\ \hat{y} + dx\ dy\ \hat{z}$

Coordenadas esféricas

$d\boldsymbol{l} = dr\ \hat{\boldsymbol{r}} + r\ d\theta\ \hat{\boldsymbol{\theta}} + r\ \text{sen}\ \theta\ d\varphi\ \hat{\boldsymbol{\varphi}}$

$d^3r = r^2\text{sen}\ \theta\ dr\ d\theta\ d\varphi$

$\boldsymbol{\nabla}\phi = \partial_r\phi\ \hat{\boldsymbol{r}} + r^{-1}\partial_\theta\phi\ \hat{\boldsymbol{\theta}} + (r\ \text{sen}\ \theta)^{-1}\partial_\varphi\phi\ \hat{\boldsymbol{\varphi}}$

$\boldsymbol{\nabla}\cdot\boldsymbol{a} = r^{-2}\partial_r(r^2 a_r) + (r\ \text{sen}\ \theta)^{-1}\partial_\theta(\text{sen}\ \theta\ a_\theta) + (r\ \text{sen}\ \theta)^{-1}\partial_\varphi a_\varphi$

$\boldsymbol{\nabla}\times\boldsymbol{a} = (r\ \text{sen}\ \theta)^{-1}[\partial_\theta(\text{sen}\ \theta\ a_\varphi) - \partial_\varphi a_\theta]\ \hat{\boldsymbol{r}} + r^{-1}[(\text{sen}\ \theta)^{-1}\partial_\varphi a_r - \partial_r(r a_\varphi)]\ \hat{\boldsymbol{\theta}}$
$\quad + r^{-1}[\partial_r(r a_\theta) - \partial_\theta a_r]\ \hat{\boldsymbol{\varphi}}$

$\nabla^2\phi = r^{-2}\partial_r(r^2\partial_r\phi) + (r^2\text{sen}\ \theta)^{-1}\partial_\theta(\text{sen}\ \theta\partial_\theta\phi) + (r^2\text{sen}^2\theta)^{-1}\partial^2_\varphi\phi$

$d\boldsymbol{S} = r^2 sen\ \theta\ d\theta\ d\phi\ \hat{r} + r\ sen\ \theta\ dr\ d\phi\ \hat{\theta} + r\ dr\ d\theta\ \hat{\phi}$

Coordenadas cilíndricas

$d\boldsymbol{l} = dr\ \hat{\boldsymbol{r}} + r\ d\varphi\ \hat{\boldsymbol{\varphi}} + dz\ \hat{\boldsymbol{z}}$

$d^3r = r\ dr\ d\varphi\ dz$

$\boldsymbol{\nabla}\phi = \partial_r\phi\ \hat{\boldsymbol{r}} + r^{-1}\partial_\varphi\phi\ \hat{\boldsymbol{\varphi}} + \partial_z\phi\ \hat{\boldsymbol{z}}$

$\boldsymbol{\nabla}\cdot\boldsymbol{a} = r^{-1}\partial_r(r a_r) + r^{-1}\partial_\varphi a_\varphi + \partial_z a_z$

$\boldsymbol{\nabla}\times\boldsymbol{a} = [r^{-1}\partial_\varphi a_z - \partial_z a_\varphi]\ \hat{\boldsymbol{r}} + [\partial_z a_r - \partial_r a_z]\ \hat{\boldsymbol{\varphi}} + r^{-1}[\partial_r(r a_\varphi) - \partial_\varphi a_r]\ \hat{\boldsymbol{z}}$

$\nabla^2\phi = r^{-1}\partial_r(r\partial_r\phi) + r^{-2}\partial^2_\varphi\phi + {\partial_z}^2\phi$

$d\boldsymbol{S} = r\ d\phi\ dz\ \hat{r} + dr\ dz\ \hat{\phi} + r\ dr\ d\phi\ \hat{z}$