Ecuación de estado del universo

Parte 2: Tomorrow never knows

et donc

$$\kappa\rho = \frac{\alpha}{R^3} + \frac{3\beta}{R^4} \tag{9}$$

Substituant dans (2), nous avons à intégrer

$$\frac{R'^2}{R^2} = \frac{\lambda}{3} - \frac{1}{R^2} + \frac{\kappa\rho}{3} = \frac{\lambda}{3} - \frac{1}{R^2} + \frac{\alpha}{3R^3} + \frac{\beta}{R^4} \tag{10}$$

ou

$$t = \int \frac{dR}{\sqrt{\frac{\lambda R^2}{3} - 1 + \frac{\alpha}{3R} + \frac{\beta}{R^2}}} \tag{11}$$

Pour α et β égaux à zéro, nous trouvons la solution de de Sitter (¹)

$$R = \sqrt{\frac{3}{\lambda}} \cosh \sqrt{\frac{\lambda}{3}} (t - t_o) \tag{12}$$

La solution d'Einstein s'obtient en posant $\beta = 0$ et R constant. Posant

Relatividad General

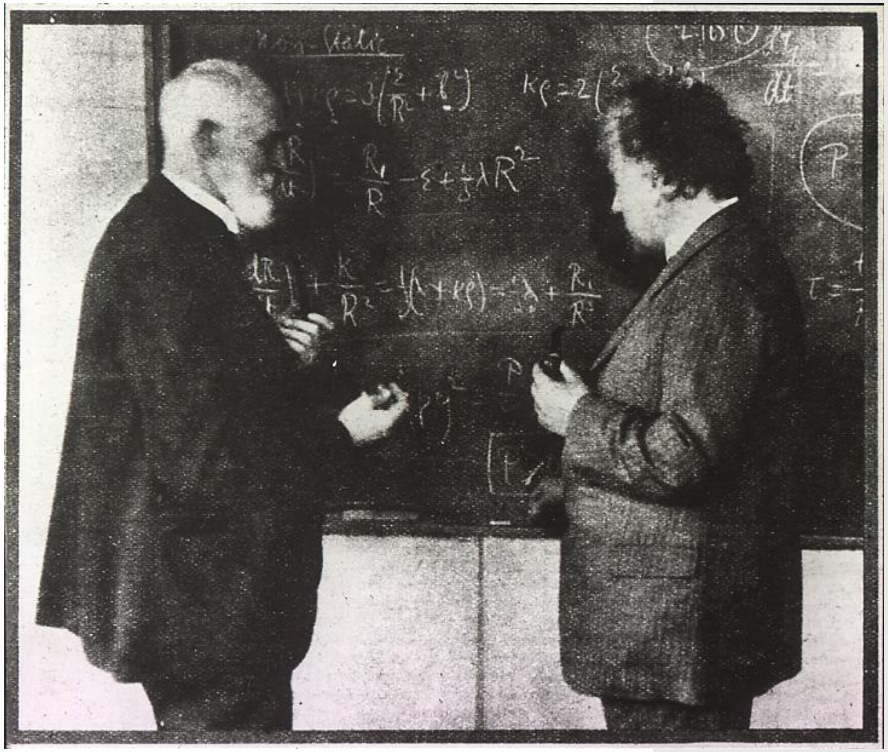
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi \frac{G T_{\mu\nu}}{c^4}$$

$T_{\mu\nu} = tensor\ energía - momentum$
$materia\ y\ energía$

$g_{\mu\nu} = tensor\ métrico$

$R_{\mu\nu} = tensor\ de\ Ricci$
$derivadas\ 2das\ de\ la\ métrica$

$R = escalar\ de\ Ricci$
$derivadas\ 2das\ de\ la\ métrica$



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Métrica-FLRW

Alexander Friedmann

Georges Lemaître

Howard P. Robertson

Arthur G. Walker

Métrica-FLRW

$$ds^2 = -c^2dt^2 + a(t)^2[d\rho^2 + S_k(\rho)^2 d\Omega^2]$$

$S_k = R sen\left(\frac{\rho}{R}\right) \quad k = 1$

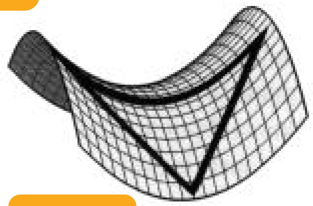
$S_k = \rho \quad k = 0$

$S_k = R senh\left(\frac{\rho}{R}\right) \quad k = -1$

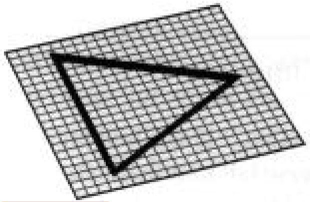
$d\Omega^2 = d\theta^2 + sen^2\theta d\phi^2$

curvatura	negativa	cero	positiva
isotrópico	si	si	si
parámetro de curvatura	-1	0	+1
suma de ángulos	<180	180	>180
área	∞	∞	4πR^2
máxima separación	∞	∞	πR
nombre	abierto	plano	cerrado

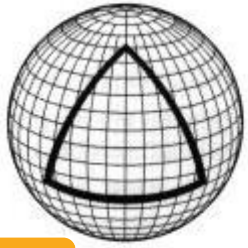
2D



k=-1



k=0



k=+1

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Ecuaciones de Friedmann

curvatura

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\varepsilon}{3c^2} - \frac{kc^2}{{R_0}^2 a(t)^2}$$ 1ra. ecuación de Friedmann

radio de curvatura en t=t_0

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3p)$$ 2da. ecuación de Friedmann

$$\mathrm{p} = \omega\varepsilon$$ ecuación de estado

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Ecuación de estado

materia no relativista

$$p = \frac{\rho}{\mu} kT \qquad p \approx \frac{kT}{\mu c^2}\varepsilon \qquad p_m = \frac{\langle v^2 \rangle}{3c^2}\varepsilon = \omega\varepsilon \to \omega \ll 1$$

radiación

$$p = \frac{1}{3}\varepsilon \to \omega = 1/3$$

Λ

$$p = -\varepsilon \to \omega = -1$$

$$\varepsilon = \sum \varepsilon_\omega \qquad p = \sum p_\omega$$

D. Dalvit, J. Frastai, I. Lawrie. Problems on Statistical Mechanics.1999

Densidad de energía

$$\dot{\varepsilon}_{\omega} + 3\frac{\dot{a}}{a}(\varepsilon_{\omega} + p_{\omega}) = 0$$

$$\frac{d\varepsilon_{\omega}}{\varepsilon_{\omega}} = -3(1+\omega)\frac{da}{a}$$

$$\varepsilon_{\omega}(\mathrm{a}) = \varepsilon_{\omega.0}a^{-3(1+\omega)}$$

materia no relativista

$$\omega = 0 \; \varepsilon_{M}(\mathrm{a}) = \varepsilon_{M.0}a^{-3}$$

radiación

$$\omega = 1/3 \; \varepsilon_{R}(\mathrm{a}) = \varepsilon_{R.0}a^{-4}$$

Λ

$$\omega = -1 \; \varepsilon_{\Lambda}(\mathrm{a}) = \varepsilon_{\Lambda.0}$$

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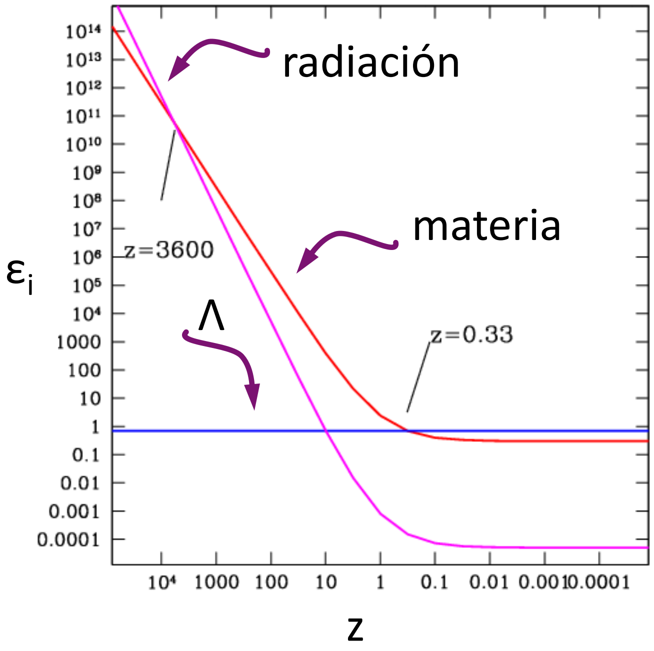
Densidad de energía



$$\frac{\varepsilon_M(a)}{\varepsilon_R(a)} = \frac{\varepsilon_{M,0}/a^3}{\varepsilon_{R,0}/a^4} = \frac{\varepsilon_{M,0}}{\varepsilon_{R,0}} a$$

$$a \approx \frac{1}{3600} \to z \approx 3600$$

$$\frac{\varepsilon_\Lambda(a)}{\varepsilon_M(a)} = \frac{\varepsilon_{\Lambda,0}}{\varepsilon_{M,0}/a^3} = \frac{\varepsilon_{\Lambda,0}}{\varepsilon_{M,0}} a^3$$

$$a \approx 0{,}75 \to z \approx 0{,}33$$

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Tipos de universos

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\sum \varepsilon_{\omega,0}a^{-1-3\omega} - \frac{kc^2}{{R_0}^2}$$

Solo curvatura

$$\dot{a}^2 = -\frac{kc^2}{{R_0}^2}$$

k=0 $\dot{a} = 0$ universo estático

k=1 prohibido

k=-1 $\dot{a} = \pm\frac{c}{R_0} \rightarrow a(t) = \frac{t}{t_0}$

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Tipos de universos

universos planos k=0

materia	$\omega = 0$	$\dot{a}^2 = \frac{8\pi G}{3c^2}\varepsilon_{M,0}a^{-1} \rightarrow a(t) = \left(\frac{t}{t_0}\right)^{2/3}$	universo Einstein-de Sitter(1932)
radiación	$\omega = 1/3$	$\dot{a}^2 = \frac{8\pi G}{3c^2}\varepsilon_{R,0}a^{-2} \rightarrow a(t) = \left(\frac{t}{t_0}\right)^{1/2}$	
Λ	$\omega = -1$	$\dot{a}^2 = \frac{8\pi G}{3c^2}\varepsilon_{\Lambda,0}a^{2} \rightarrow a(t) = \exp[\frac{1}{2}\left(\frac{t-t_0}{t_0}\right)]$	

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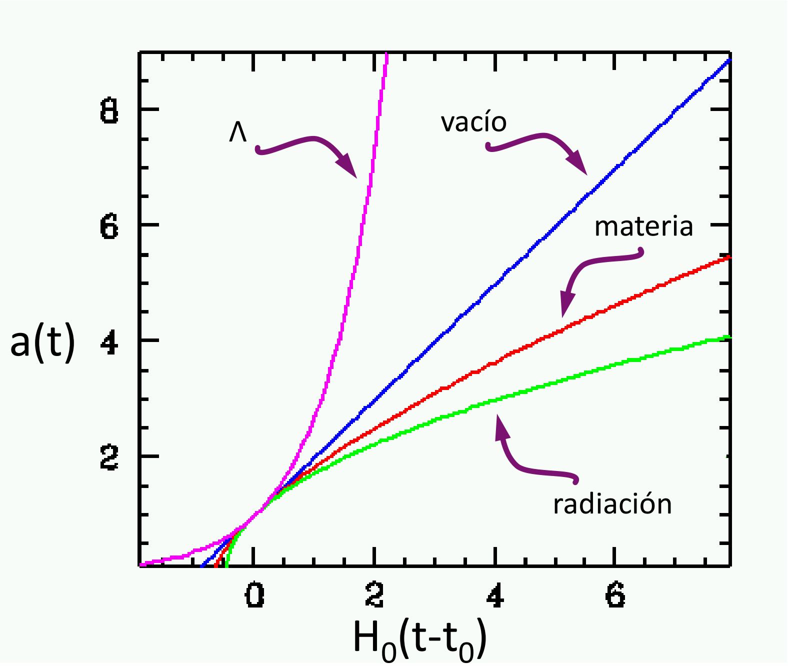
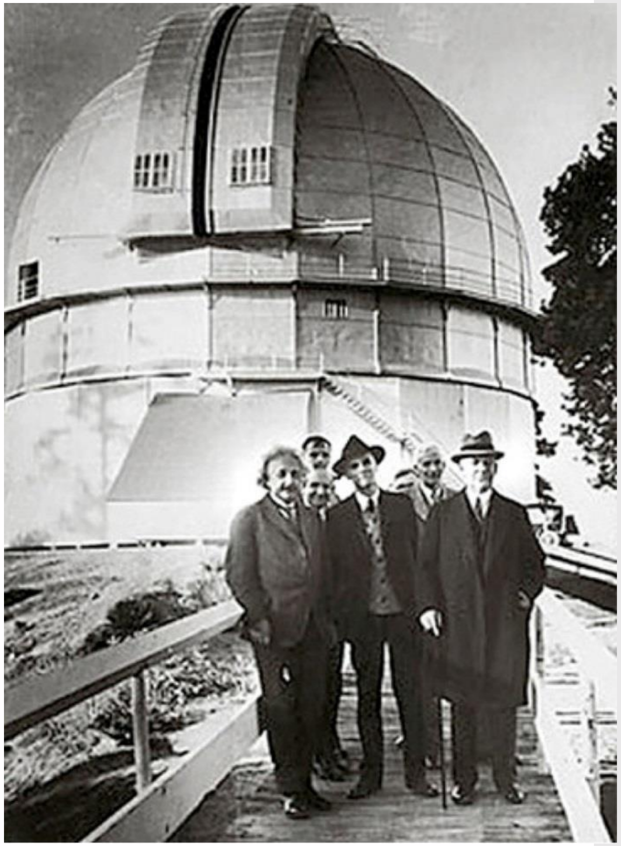
Tipos de universos





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