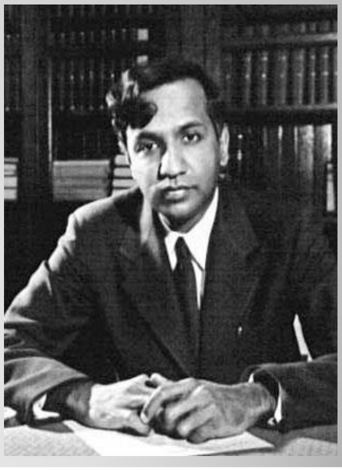
Límite de chandrasekhar y evolución estelar

Historia

Arthur Edington
(1882-1944)
Corroboro la teoría de la relatividad de Einstein
Máxima figura de la astronomía inglesa

Subrahmanyan Chandrasekhar
(1910-1995)
1930-Trinity college de la Universidad de Cambridge
"The Internal Constitution of Stars"

Limite de Chandrasekhar
1.4 Masas solares
Nobel 1983



Tipos de estrellas

- Secuencia principal
- Enanas blancas
- Estrella de neutrones

$0.1M_{\odot} < M < 100M_{\odot}$
$R_{\odot} = 7 \times 10^{8}m$
$\langle\rho\rangle = 1.4\ g.cm^{-3}$
$M_{\odot} = 2 \times 10^{30}Kg$
$L_{\odot} = 3.8 \times 10^{26}W$
$T_{s,\odot} = 6000K$
$\langle n_p\rangle \approx 1 \times 10^{57}$

$0.5M_{\odot} < M < 1.4M_{\odot}$
$R \approx 5000Km$
$\rho \approx 1 \times 10^{6}\ g.cm^{-3}$

$M \approx 1.4M_{\odot}$
$R \approx 10km$
$\rho \approx 3 \times 10^{14}\ g.cm^{-3}$

Ecuaciones de estado

$$\Omega = -\int d\varepsilon \mathcal{N}(\varepsilon)\, f(\varepsilon)$$

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/kT} - \eta}$$

$$N = \int d\varepsilon \mathfrak{D}(\varepsilon) f(\varepsilon),\, U = \int d\varepsilon \mathfrak{D}(\varepsilon)\varepsilon f(\varepsilon) \qquad \mathfrak{D}(\varepsilon) = \frac{\partial \mathcal{N}}{\partial \varepsilon}$$

Partículas no relativistas

$$\mathcal{N}(\varepsilon) = \frac{2s+1}{h^3} V \int d^3\mathbf{p}\,\theta\left(\varepsilon - \frac{p^2}{2m}\right) = \frac{V}{3\pi^2\hbar^3}(2m\varepsilon)^{3/2}$$

$$\frac{U}{N} = \frac{3}{5}\varepsilon_F, \varepsilon_F = \frac{p_F^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

$$P = \frac{2}{3}\frac{U}{V}$$

Partículas relativistas

$$\mathcal{N}(\varepsilon) = \frac{8\pi V}{3h^3c^3}\varepsilon^3$$

$$P = \frac{1}{3}\frac{U}{V}$$

- Cálculo del radio de las estrellas y problemas de estabilidad

$\varepsilon_f \approx 100MeV \quad (1eV = 11600K)$

$T \approx 10^8\ K$

$kT/\varepsilon_f \approx 10^{-4}$

$m_n c^2 \approx 900\ MeV \gg \varepsilon_f$

$E = U + E_G \qquad m_n \int d^3\mathbf{r}\, n(r) = \int d^3\mathbf{r}\, \rho(r) = M$

$$U = \frac{3(3\pi^2)^{2/3}\hbar^2}{10 m_n} \int d^3\mathbf{r} [n(r)]^{5/3}$$

$$E_G = -\frac{G}{2} \int d^3\mathbf{r} d^3\mathbf{r}' \frac{\rho(r)\rho(r')}{|\mathbf{r} - \mathbf{r}'|}$$

$E_G = -2U, E = -U \qquad R = \left(\frac{9\pi}{4}\right)^{2/3} \frac{\hbar^2}{G m_n^{8/3} M^{1/3}}$

- Cálculo estimado: $M = M_\odot \qquad \varepsilon_f = 60MeV \qquad R = 12Km$

$$U = \frac{3^{4/3}hc}{8\pi^{1/3}} \int d^3\mathbf{r} [n(r)]^{4/3} \simeq \frac{3^{5/3}hcM^{4/3}}{2^{11/3}\pi^{2/3} m_n^{4/3}} \frac{1}{R}$$

$$M > \frac{3}{16\pi m_n^2} \left(\frac{5hc}{2G}\right)^{3/2} \simeq 7M_\odot$$

$$\varepsilon_f = \frac{1}{2}\frac{GMm_n}{R}$$

$$\varepsilon - \frac{GMm_n}{R}$$

$$\left(\frac{2GM}{R}\right)^{1/2} \simeq 1.5 \times 10^8 \left(\frac{M}{M_\odot}\right)^{2/3} ms^{-1}$$

Maximización de la entropía

- Relación entre masa, radio y temperatura

$$S-\beta(U+E_G) \qquad \beta=T^{-1} \qquad 3PV=-E_G=\frac{3GM^2}{5R}$$

$$\frac{dP}{dV_s}<\frac{d^2E_G}{dV^2}=\frac{4}{9}\frac{E_G}{V^2}=-\frac{4}{3}\frac{P}{V} \qquad k_s\equiv-\frac{P}{V}\left(\frac{\partial P}{\partial V}\right)_s$$

$$k_s=\frac{3}{5}\ (clasico\ o\ cuantico) \qquad E=U+E_G=\frac{1}{2}E_G=-U=-\frac{3}{2}PV$$

$$k_s=\frac{3}{4}\ (ultrarelativista) \qquad E=3PV+E_G=0$$

- Nube de hidrógeno como gas ideal:

$$-E=U=\frac{3}{2}NkT=C_VT \qquad N=\frac{2M}{m_p} \qquad R=\frac{Gm_p}{10k}\frac{M}{T}$$

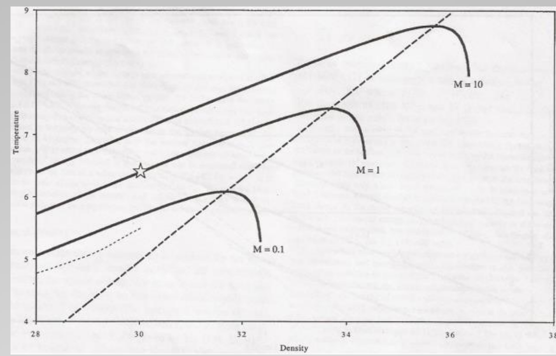
$$e^{\mu_s/kT}=\frac{n}{2}\left(\frac{2\pi\rho^2}{m_s kT}\right)^{3/2}\ll 1 \qquad Para\ el\ sol: T\approx 2\times 10^6 K$$

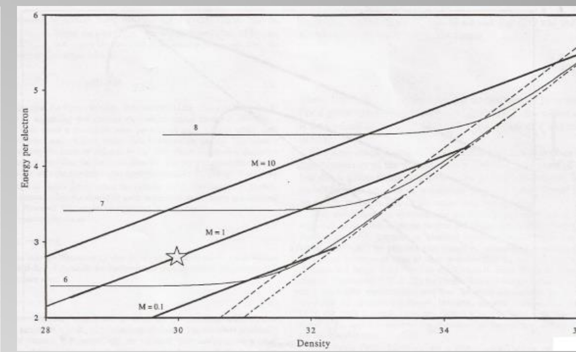
Maximización de la entropía

$$\mu_e \equiv kt\alpha$$

$$n_e \equiv \frac{N_e}{V} = \frac{(2m_e kT)^{3/2}}{12\pi^2\hbar^3} I_{3/2}(\alpha)$$

$$-\frac{E}{3kT} = \frac{GM^2}{10RkT} = \frac{N_e}{5}\frac{I_{5/2}(\alpha)}{I_{3/2}(\alpha)} + \frac{N_{nuc}}{2}$$





- Densidad Local

$$\delta\omega_i = \omega_i \nabla\cdot\delta\boldsymbol{r}$$

$$\delta E_G = \int d^3\boldsymbol{r}\,\rho(r)\nabla W\cdot\delta\boldsymbol{r}$$

$$W(r) = -G\int \frac{d^3\boldsymbol{r}'\rho(r')}{|\boldsymbol{r}-\boldsymbol{r}'|} = -\frac{4\pi G}{r}\int_0^r r'^2 dr'\rho(r') - 4\pi G\int_r^\infty r' dr'\rho(r')$$

$$\delta S = \sum_i\left[\frac{\delta u_i}{T_i} + \frac{P_i}{T_i}\delta\omega_i\right] = \sum_i \frac{\delta u_i}{T_i} + \int d^3\boldsymbol{r}\frac{P}{T}\nabla\cdot\delta\boldsymbol{r} \qquad \nabla P = \rho\nabla W \qquad \nabla\cdot\frac{\nabla P}{\rho} = -4\pi G\rho \qquad 3\int d^3\boldsymbol{r}P = -E_G$$

Luz emitida por una estrella

- Una estrella como cuerpo negro y cinética de los fotones en la materia

$$\partial w = L_v(\theta) cos\theta \, \partial S \partial w \partial v \qquad L_v^0(T) = \frac{2hv^3}{c^2}\frac{1}{e^{hv/kT}-1}$$

$$L = 4\pi R^2 \sigma T_s^4, \sigma = \frac{\pi^2 k^4}{60\hbar^3 c^2} = 5.67 \times 10^{-8}\, W m^{-2} K^{-4}$$

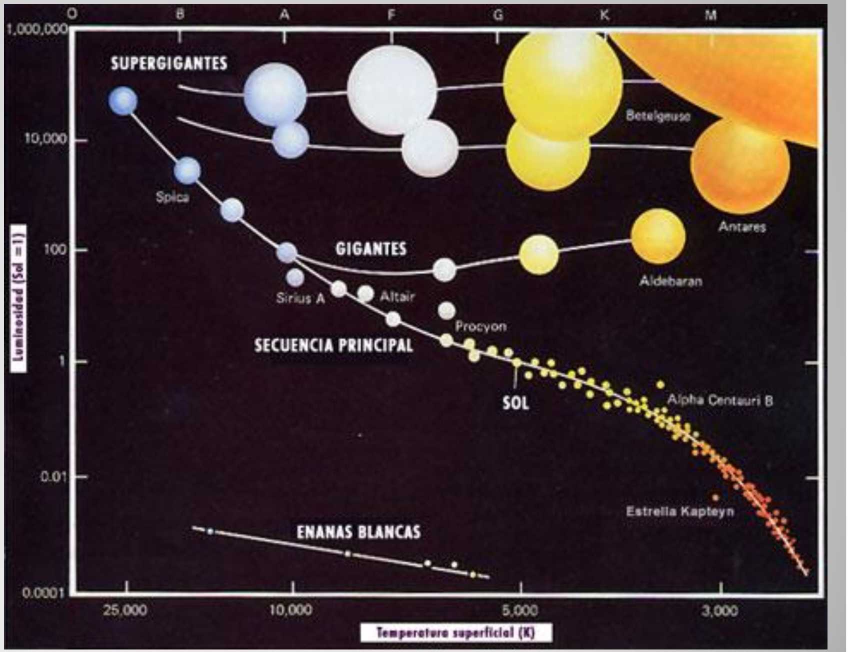
$$\frac{\partial f}{\partial t} + v.\nabla f = I(f)$$

$$\Lambda_v(r,u,t) \equiv \frac{h^4 v^3}{c^2} f(r,p,t)$$

$$\frac{\partial \Lambda_v}{\partial t} + cu \cdot \nabla \Lambda_v = I_c + I_{sp} + I_{st} - I_a$$

$$\langle N_\gamma \rangle = \frac{1}{2} h^3 f = \frac{c^2}{2hv^3} \Lambda_v$$

$$\frac{1}{c}\frac{\partial \Lambda_v}{\partial t} + u \cdot \nabla \Lambda_v = k_v (L_v^0 - \Lambda_v) \qquad k_v = \sum \sigma_a (n_1 - n_2)$$



Luz emitida por una estrella

- Cinética de los fotones en la materia y radiación de cuerpo negro:

$$\frac{dA_v}{dz} = \frac{k_v}{\cos\theta}(L_v^0 - A_v) \qquad \zeta \equiv \int_z^\infty dz' k_v(z')$$

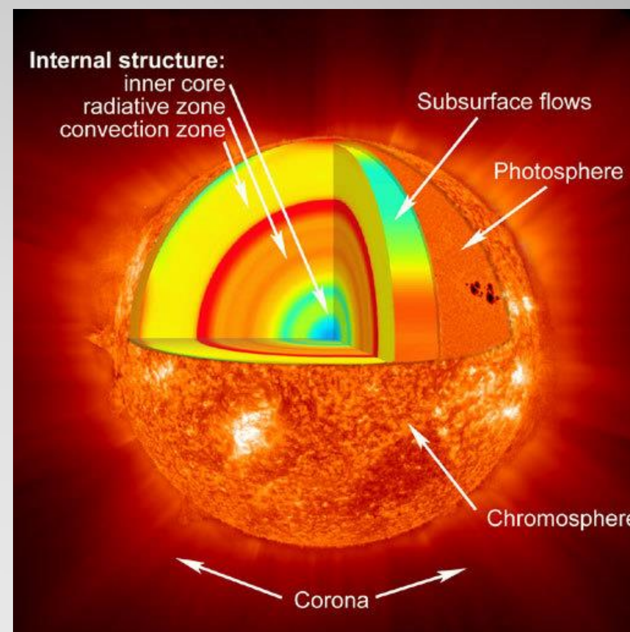
$$\frac{dA_v}{d\zeta} = \frac{A_v - L_v^0}{\cos\theta}$$

$$A_v(\zeta,\theta) = \int_0^\infty \frac{d\zeta'}{\cos\theta} e^{-\zeta'/\cos\theta} L_v^0(\zeta+\zeta'),\ (0<\theta<\pi/2)$$

$$A_v(\zeta,\theta) = \int_0^\zeta \frac{d\zeta'}{|\cos\theta|} e^{-\zeta'/|\cos\theta|} L_v^0(\zeta-\zeta'), (\pi/2<\theta<\pi)$$

$$L_v(\theta) = \int_0^\infty \frac{d\zeta}{cos\theta} e^{-\zeta/cos\theta} L_v^0[T(z)] \simeq L_v^0\left[T\left(-\frac{cos\theta}{k_v}\right)\right]$$

$\zeta: 0 \to 1\,; R \approx 20km$ $\qquad \zeta: 0 \to 10\,; R \approx 500km$



$R_{fotosfera} \approx 300Km$

$R_{cromosfera} \approx 1500Km$

$R_{\odot} = 700000Km$

Transferencia de radiación difusión y movimiento browniano

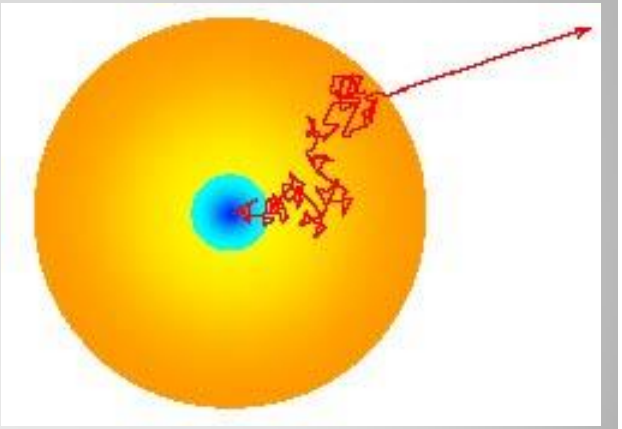
- Ecuaciones de transporte y camino aleatorio de los fotones

$$\boldsymbol{J}(\boldsymbol{r}) = \int d^3\boldsymbol{p}\, c\boldsymbol{u}\, f(\boldsymbol{r},\boldsymbol{p})cp = \int d^2\boldsymbol{u}\, d\nu\, \boldsymbol{u}\Lambda_\nu(\boldsymbol{r},\boldsymbol{u})$$

$$\frac{\partial u}{\partial t} + \nabla \cdot \boldsymbol{J} = q$$

$$u_\gamma(T) = \int d^3\boldsymbol{p}\, f(\boldsymbol{r},\boldsymbol{p})cp \simeq \frac{1}{c}\int d^2\boldsymbol{u}\, d\nu\, L_\nu^0(\boldsymbol{r}) = \frac{4}{c}\sigma T^4$$

$$\boldsymbol{J} = -\frac{c}{3\rho\kappa}\nabla u_\gamma \qquad l = \frac{1}{\rho\kappa}$$



$$\boldsymbol{J} = -\frac{c}{3n_e\sigma_{Th}}\nabla u_\gamma \qquad k = \frac{\sigma_{Th}}{m_p} = 4\times 10^{-2} m^2 kg^{-1}$$

$$\frac{\partial u_\gamma}{\partial t} - \frac{c}{3\rho\kappa}\nabla^2 u_\gamma = q \qquad u_\gamma(r,t) = \left(\frac{3\rho\kappa}{4\pi ct}\right)^{3/2} exp\left[-\frac{3\rho\kappa r^2}{4ct}\right]$$

$$\langle r^2\rangle = \left(\frac{2ct}{\rho k}\right)t \qquad Sea : \rho_{sol} = 1.4\times 10^3\ kg.m^{-3} \qquad \langle r^2\rangle = R_\odot^2\ toma\ 1500\ \text{años}$$

$$Largo\ de\ cada\ paso = l\sqrt{2} \qquad \langle r^2\rangle = 2l^2 n = 2lct \quad l = 1.8\ cm \qquad n_{colisiones} = 8\times 10^{20}$$

Transferencia de radiación difusión y movimiento browniano

- La temperatura dentro de la estrella y la relación entre masa y luminosidad

$$\frac{d}{dr}\left(\frac{\sigma r^2}{\rho\kappa}\frac{dT^4}{dr}\right) = -\frac{3r^2}{4}q$$

$$T^4(r) = T^4(0) - \frac{\rho\kappa q}{8\sigma}r^2 \; (r < R_c)$$

$$T^4(r) = T^4(0) - \frac{\rho\kappa q}{8\sigma}\left(3R_c^2 - 2\frac{R_c^3}{r}\right) \; (r > R_c)$$

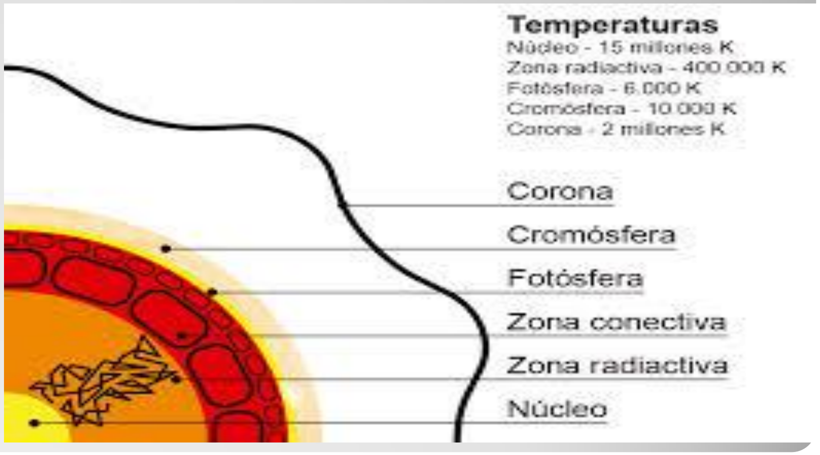
$$Para\ el\ sol:\ R_c = \frac{1}{15}R_\odot \quad L_\odot = Q = \frac{4}{3}\pi R_c^3 q$$

$$T(0) \simeq \left[\frac{9\rho k Q}{32\pi\sigma R_c}\right]^{1/4} \simeq 5\times 10^6 K$$

$$L = -\frac{16\pi\sigma r^2}{3\rho\kappa}\frac{dT^4}{dr}$$

$$L = -\frac{32\pi\sigma R}{9\rho k}\left(\frac{Gm_pM}{10kR}\right)^4 = \frac{7.7\times 10^{-4}G^4m_p^4}{\hbar^3c^2\kappa}M^3 \simeq \frac{10^{-66}}{\kappa}M^3$$

$$Sea\ M = M_\odot \qquad L = 2.2\times 10^{26}W \qquad L_\odot = 3.8\times 10^{26}W$$



Combustible nuclear

- Reacciones nucleares y el pico de Gamow

$$p + p \rightarrow d + e^+ + \nu + 0.4\ MeV$$

$$e^+ + e^- \rightarrow \gamma + 1.0\ MeV$$

$$d + p \rightarrow {}^3He + \gamma + 5.5\ MeV$$

$${}^3He + {}^3He \rightarrow {}^4He + 2p + 12.9\ MeV$$

$$4p + 2e^- \rightarrow {}^4He + 2\nu + 26.7\ MeV$$

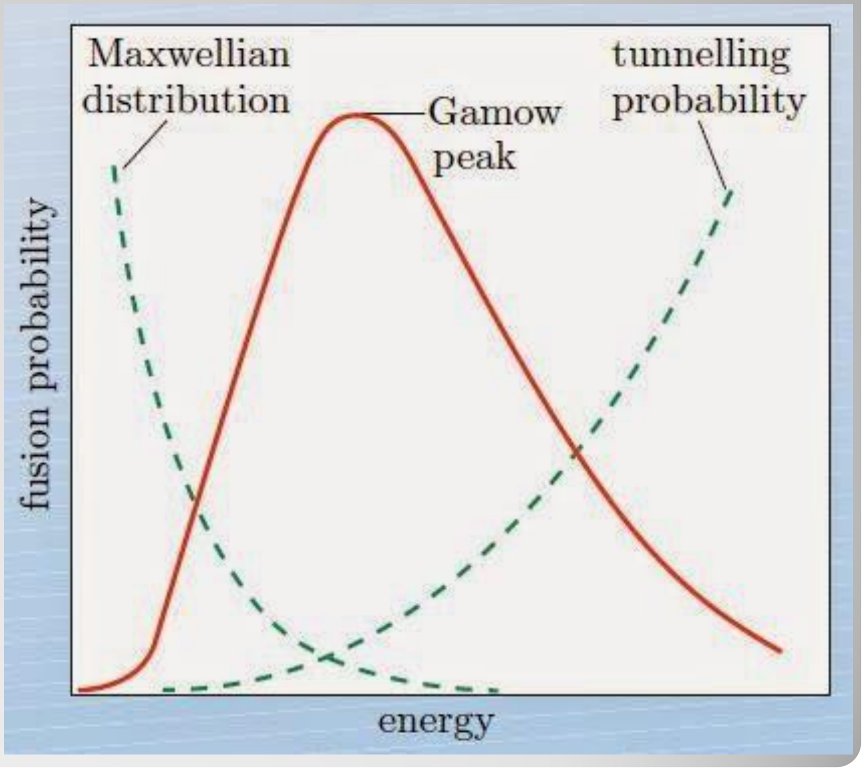
$$\sigma = \frac{S(\varepsilon)}{\varepsilon} exp\left[-\frac{2\sqrt{2\mu}}{\hbar}\int\left(\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r} - \varepsilon\right)^{1/2} dr\right]$$

$$\sigma = \frac{S(\varepsilon)}{\varepsilon} \exp\left(-\sqrt{\varepsilon_B/\varepsilon}\right), \varepsilon_B = \frac{\mu Z_1^2 Z_2^2 e^4}{8\hbar^2 \epsilon_0^2}$$

$$\varepsilon = \frac{1}{2}\mu(V_1 - V_2)^2 \qquad \mu = \frac{m_1 m_2}{(m_1 + m_2)}$$

$$|V_1 - V_2| n_1 \left(\frac{m_1}{2\pi kT}\right)^{3/2} e^{-m_1 V_1^2/2kT} d^3 V_1$$

$$w = \frac{4 n_1 n_2}{(2\pi\mu)^{1/2}(kT)^{3/2}} \int d\varepsilon S(\varepsilon) \exp\left[-\sqrt{\varepsilon_B/\varepsilon} - \varepsilon/kT\right]$$



Combustible nuclear ec de balance

$$\varepsilon_G = \left(\frac{kT}{2}\right)^{2/3} \varepsilon_B^{1/3} \qquad w = \frac{4(2\varepsilon_B)^{1/6}}{(3\mu)^{1/2}(kT)^{2/3}} n_1 n_2 S(\varepsilon_G) exp\left[-3\left(\frac{\varepsilon_B}{4kT}\right)^{1/3}\right]$$

Las reacciones suceden entre: $\varepsilon_G \pm (kT\varepsilon_G)^{1/2}$

Para el sol: $n_1 = n_2 = 10^{32} m^{-3}$ $\quad kT = 1\, keV \quad S = 3.8 \times 10^{-50} m^2 .keV \quad w = 2 \times 10^{14} m^{-3}. s^{-1}$

- Producción de potencia y la estabilidad del equilibrio estelar

$q = \eta w \simeq 500\, Wm^{-3} \simeq 4 \times 10^{-3}\, Wkg^{-1}$ $\qquad q_{Humano} \simeq 1.5\, W. kg^{-1}$

$$Q = \frac{4}{3}\pi R_c^3 q \simeq 2 \times 10^{26}\, W$$

$\frac{\eta}{2}\frac{M_\odot}{10 m_p}\frac{1}{L_\odot} = 10^{10}$ *años* $\qquad$ *Tiempo de reaccion de proton en nucleo:* $\frac{1}{2} n w^{-1} \approx 8 \times 10^9$ *años*

Ecuacion balance dinamico: $\frac{dE}{dt} = Q - L$

síntesis

- $L \simeq M^3$

  $E_{nuclear} \simeq M$

  $t_{1/2} \simeq \frac{E_{nuclear}}{L} \simeq \frac{1}{M^2}$