

Ecuación de estado del universo

The background is a vibrant, abstract composition. It features a large yellow circle on the left, a blue circle on the right, and a pink circle at the bottom. There are also several smaller circles and lines in various colors (yellow, blue, pink, purple). In the foreground, there are four black silhouettes of people walking from left to right on a pink path. The overall style is modern and artistic.

Parte 1: Across the universe



et donc

$$\kappa\rho = \frac{\alpha}{R^3} + \frac{3\beta}{R^4} \quad (9)$$

Substituant dans (2), nous avons à intégrer

$$\frac{R'^2}{R^2} = \frac{\lambda}{3} - \frac{1}{R^2} + \frac{\kappa\rho}{3} = \frac{\lambda}{3} - \frac{1}{R^2} + \frac{\alpha}{3R^3} + \frac{\beta}{R^4} \quad (10)$$

ou

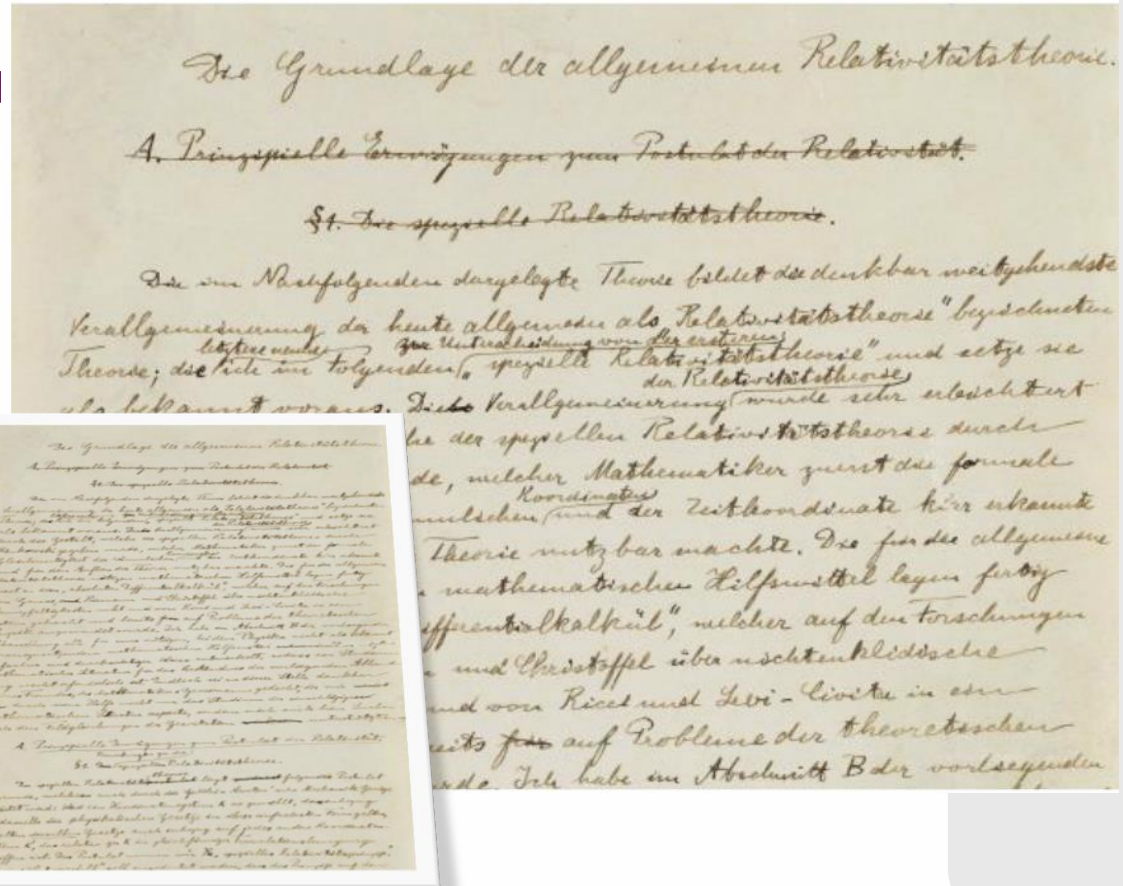
$$t = \int \frac{dR}{\sqrt{\frac{\lambda R^2}{3} - 1 + \frac{\alpha}{3R} + \frac{\beta}{R^2}}} \quad (11)$$

Pour α et β égaux à zéro, nous trouvons la solution de de Sitter (¹)

$$R = \sqrt{\frac{3}{\lambda}} \cosh \sqrt{\frac{\lambda}{3}} (t - t_0) \quad (12)$$

La solution d'Einstein s'obtient en posant $\beta = 0$ et R constant. Posant

1916-Einstein



1917-Einstein

144 Sitzung der physikalisch-mathematischen Klasse vom 8. Februar 1917
der an sich nicht beansprucht, ernst genommen zu werden; er dient
nur dazu, das Folgende besser hervortreten zu lassen. An die Stelle
der Poissonschen Gleichung setzen wir

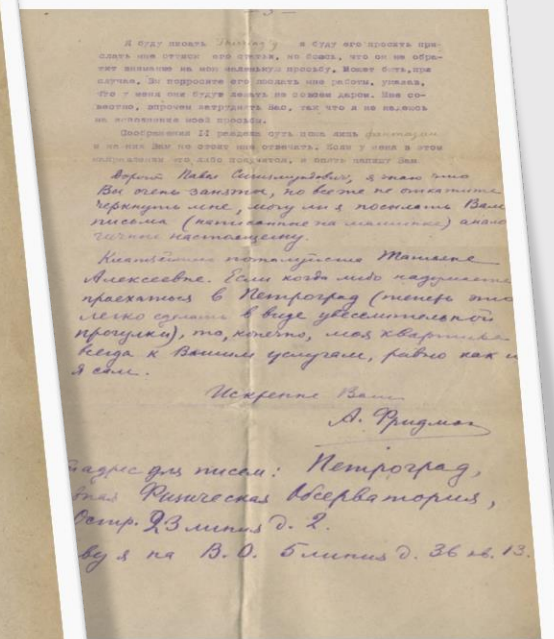
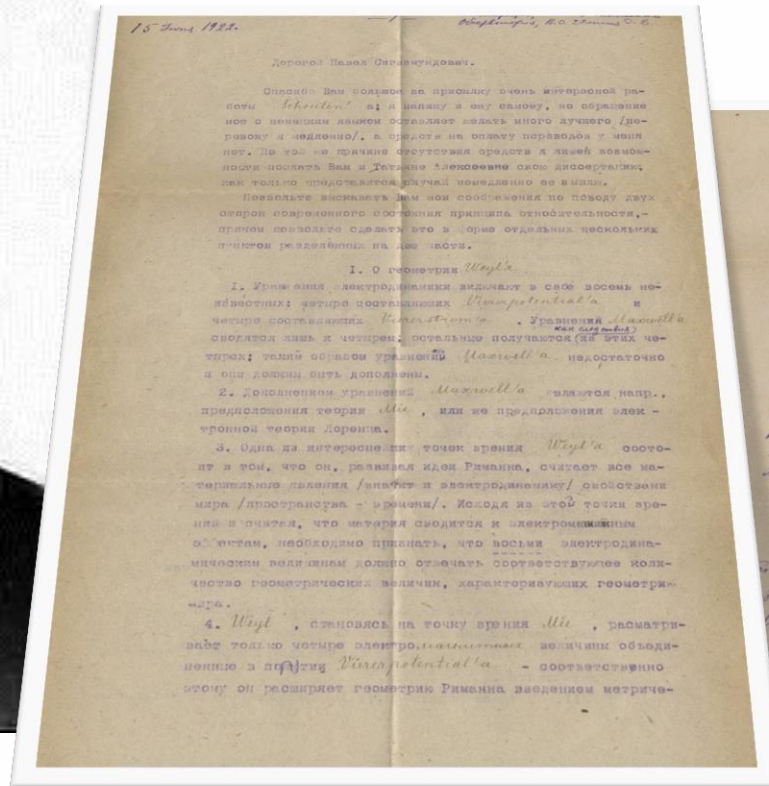
$$\Delta\phi - \lambda\phi = 4\pi K\rho, \quad (2)$$

wobei λ eine universelle Konstante bedeutet. Ist ρ_0 die (gleichmäßige)
Dichte einer Massenverteilung, so ist

$$\phi = -\frac{4\pi K}{\lambda}\rho_0 \quad (3)$$

eine Lösung der Gleichung (2). Diese Lösung entspräche dem Falle,
daß die Materie der Fixsterne gleichmäßig über den Raum verteilt
wäre, wobei die Dichte ρ gleich ρ_0 ist.

1922-A. Friedmann



1924-A. Friedmann

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Über die Krümmung des Raumes.

Von A. Friedmann in Petersburg.

Mit einer Abbildung. (Eingegangen am 29. Juni 1922.)

§ 1. 1. In ihren bekannten Arbeiten über allgemeine kosmologische Fragen kommen Einstein¹⁾ und de Sitter²⁾ zu zwei möglichen Typen des Weltalls; Einstein erhält die sogenannte Zylinderwelt, in der der Raum³⁾ konstante, von der Zeit unabhängige Krümmung besitzt, wobei der Krümmungsradius verbunden ist mit der Gesamtmasse der im Raume vorhandenen Materie; de Sitter erhält eine Kugelwelt, in welcher nicht nur der Raum, sondern auch die Welt in gewissem Sinne als Welt konstanter Krümmung angesprochen werden kann⁴⁾. Dabei werden wie von Einstein so auch von de Sitter gewisse Voraussetzungen über den Materietensor gemacht, die der Inkohärenz der Materie und ihrer relativen Ruhe entsprechen, d. h. die Geschwindigkeit der Materie wird als genügend klein vorausgesetzt im Vergleich zu der Grundgeschwindigkeit⁵⁾ — der Lichtgeschwindigkeit.

Das Ziel dieser Notiz ist, erstens die Ableitung der Zylinder- und Kugelwelt (als spezielle Fälle) aus einigen allgemeinen Annahmen, und zweitens der Beweis der Möglichkeit einer Welt, deren Raumkrümmung konstant ist in bezug auf drei Koordinaten, die als Raumkoordinaten gelten, und abhängig von der Zeit, d. h. von der vierten — der Zeitkoordinate; dieser neue Typus ist, was seine übrigen Eigenschaften anbetrifft, ein Analogon der Einsteinschen Zylinderwelt.

2. Die Annahmen, die wir unseren Betrachtungen zugrunde legen, zerfallen in zwei Klassen. Zu der ersten Klasse gehören Annahmen, welche mit den Annahmen Einsteins und de Sitters zusammen-

¹⁾ Einstein, Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie, Sitzungsberichte Berl. Akad. 1917.

²⁾ de Sitter, On Einstein's theory of gravitation and its astronomical consequences. Monthly Notices of the R. Astronom. Soc. 1916—1917.

³⁾ Unter „Raum“ verstehen wir hier einen Raum, der durch eine Mannigfaltigkeit von drei Dimensionen beschrieben wird; der „Welt“ entspricht eine Mannigfaltigkeit von vier Dimensionen.

⁴⁾ Klein, Über die Integralform der Erhaltungssätze und die Theorie der räumlich-geschlossenen Welt. Götting. Nachr. 1918.

⁵⁾ Siehe diesen Namen bei Eddington in seinem Buche: Espace, Temps et Gravitation, 2 Partie, S. 10. Paris 1921.

A. Friedmann,

ziehen sich auf die Gleichungen, denen die Gravitationspotentiale, und auf den Zustand und die Bewegung der zweiten Klasse gehören Annahmen über den allgemeinen metrischen Charakter der Welt; aus unserer Hypothese resultiert die Zylinderwelt Einsteins und auch die Kugelwelt. Die Typen der ersten Klasse sind die folgenden: Gravitationspotentiale genügen dem Einsteinschen Ansatz mit dem kosmologischen Gliede, das man auch gleich

$$g_{ik} \ddot{x}^k + \lambda g_{ik} = -\kappa T_{ik} \quad (i, k = 1, 2, 3, 4), \quad (A)$$

Gravitationspotentiale, T_{ik} der Materietensor, κ — eine Konstante; R_{ik} ist bestimmt durch die Gleichungen

$$R_{ik} = \frac{\partial^2 \sqrt{g}}{\partial x^i \partial x^k} - \frac{\partial}{\partial x^a} \left\{ \frac{\partial \sqrt{g}}{\partial x^a} \right\} + \left\{ \frac{\partial \sqrt{g}}{\partial x^a} \right\} \left\{ \frac{\partial \sqrt{g}}{\partial x^a} \right\}, \quad (B)$$

wo $i, k = 1, 2, 3, 4$ die Weltkoordinaten, und $\left\{ \frac{\partial \sqrt{g}}{\partial x^a} \right\}$ die Symbole zweiter Art¹⁾.

Die Welt ist inkohärent und in relativer Ruhe; oder, weniger genau, die relativen Geschwindigkeiten der Materie sind im Vergleich zu der Lichtgeschwindigkeit. In beiden Fällen ist der Materietensor durch die Gleichungen

$$T_{ik} = 0 \quad \text{für } i \text{ und } k \text{ nicht} = 4, \quad (C)$$

bestimmt, wobei c die Grundgeschwindigkeit; die Weltkoordinaten eingeteilt in drei Raumkoordinaten und die Zeitkoordinate x_4 .

Die Typen der zweiten Klasse sind die folgenden:

Die drei Raumkoordinaten x_1, x_2, x_3 haben eine konstante Krümmung, die aber abhängig durch die Zeitkoordinate ist. Das Intervall²⁾ ds , bestimmt durch die Einführung geeigneter Raumkoordinaten x_1, x_2, x_3 wird

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + \sin^2 x_1 \sin^2 x_2 dx_3^2 + 2g_{14} dx_1 dx_4 + 2g_{24} dx_2 dx_4 + g_{44} dx_4^2.$$

Die Krümmung R_{ik} und von \ddot{x}^k ist bei uns von dem üblichen ver-

schieden. Espace, Temps et Gravitation, 2 Partie. Paris 1921.

1927-G. Lemaître



UN UNIVERS HOMOGENE DE MASSE CONSTANTE ET DE RAYON CROISSANT,
RENDANT COMPTE
DE LA VITESSE RADIALE DES NÉBULEUSES EXTRA-GALACTIQUES
Note de M. l'Abbé G. LEMAITRE

1. GÉNÉRALITÉS.
La théorie de la relativité fait prévoir l'existence d'un univers homogène où non seulement la répartition de la matière est uniforme, mais où toutes les sections de l'espace sont identiques. Il s'agit sans doute d'un

— 284 —
des de fois plus grand que la distance des objets les plus éloignés
graphiés dans nos télescopes (1).
deux solutions ont donc leurs avantages. L'une s'accorde avec
l'observation des vitesses radiales des nébuleuses, l'autre tient compte de
l'abondance de la matière et donne une relation satisfaisante entre le rayon
de l'univers et la masse qu'il contient. Il semble désirable d'obtenir une
solution intermédiaire qui pourrait combiner les avantages de chacune

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UN UNIVERS HOMOGENE DE MASSE CONSTANTE ET DE RAYON CROISSANT, RENDANT COMPTE DE LA VITESSE RADIALE DES NÉBULEUSES EXTRA-GALACTIQUES

Note de M. l'Abbé G. LEMAITRE

1. GÉNÉRALITÉS.

La théorie de la relativité fait prévoir l'existence d'un univers homogène

de soleil du 16 juin de la même année : και επι τον μεν της αρχης της εμπροσθεν
χρόνον ασφαλέστατα ετηρήσαμεν, « nous avons observé avec grande précision l'heure
du premier contact » [éd. Bâle, p. 332]. Il est inutile d'observer que tout ce qui peut se
dire sur Théon, a fatalement un caractère provisoire, tant que le travail d'édition ne sera
pas terminé. Le texte de Pappus est établi ; on peut donc aborder les questions qui s'y
rapporment avec quelque chance de les résoudre.

(1) Nous considérons l'espace simplement elliptique, c'est-à-dire sans antipodes.

XLVII, A

4

vol. 2, p. 37, 1925. Pour la discussion de la partition de de
ometrical note on de Sitter's world. *Phil. Mag.* (6), vol. 47,
formé d'hyperplans normaux à une droite temporelle décrite
trajectoires des nébuleuses sont les trajectoires orthogonales
généralement plus des géodésiques et elles tendent à devenir
lorsqu'on s'approche de l'horizon du centre, c'est-à-dire de

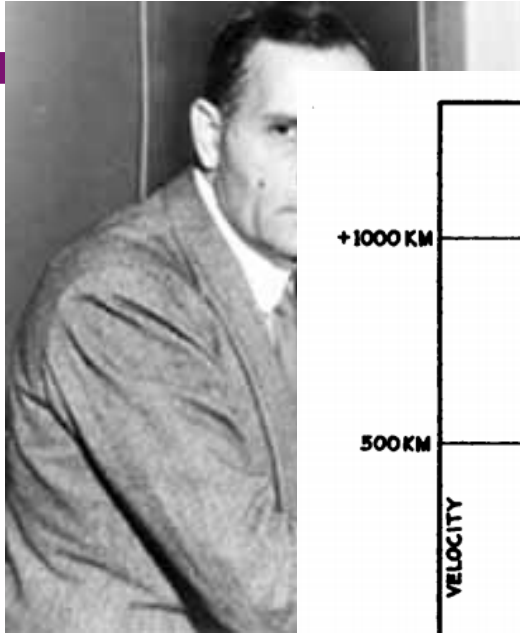
G. Lemaître. Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Annales de la Société Scientifique de Bruxelles*, A47, p. 49-59

Su física es espantosa



“Sus cálculos son correctos pero su física es espantosa”

1929-E. Hubble



A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY
AMONG EXTRA-GALACTIC NEBULAE

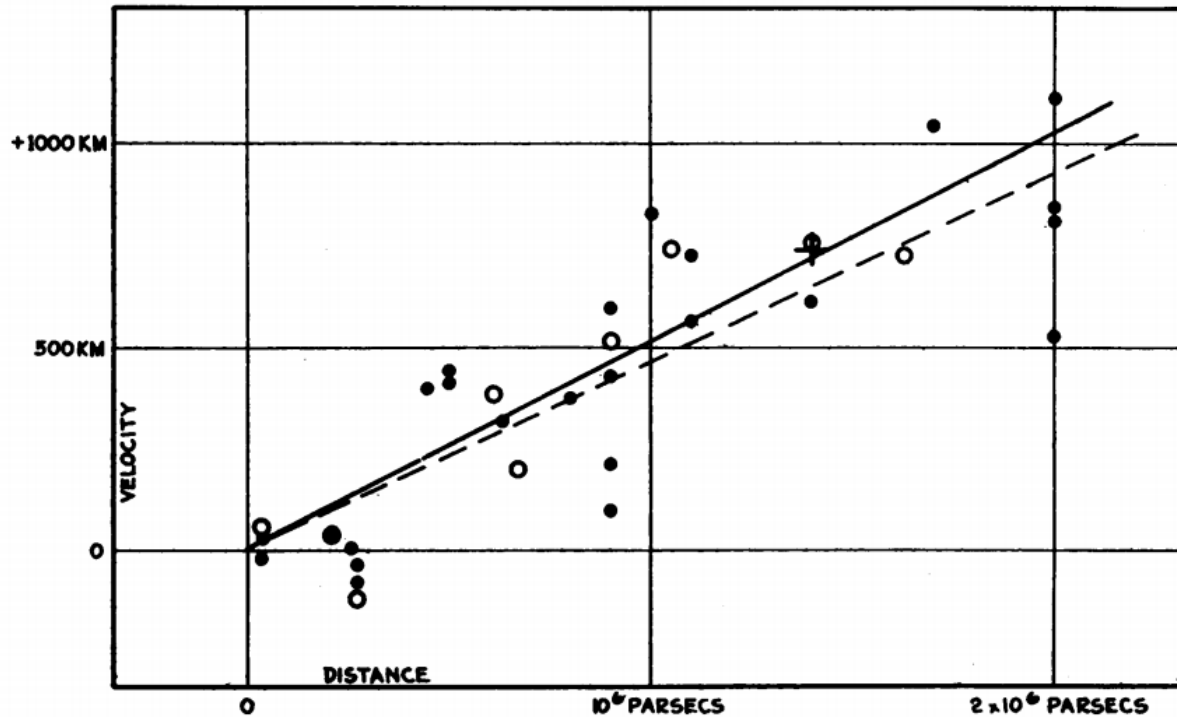
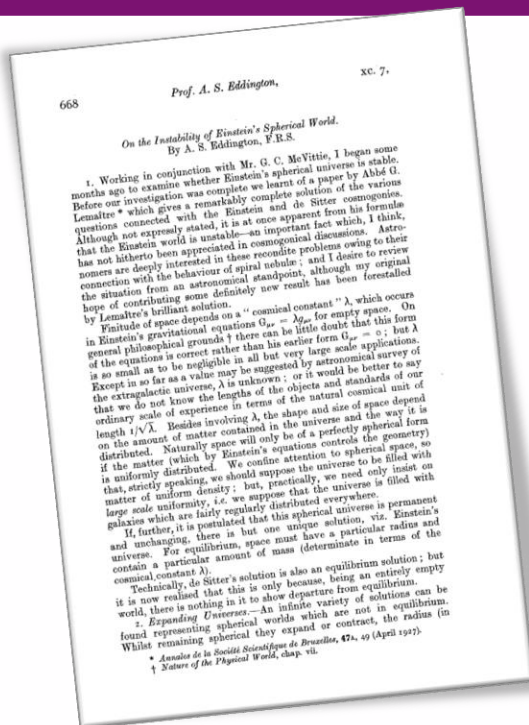


FIGURE 1

Hubble, E. P. A relation between distance and radial velocity among extra-galactic nebulae. Proc. Natl. Acad. Sci. USA, 15, 168 (1929)

A. S. Eddington



Thus both masses may diminish a little in the course of time. However, these are rather insignificant complications. In what follows we shall generally take $p = 0$, so that proper mass and relative mass are the same, and both will be conserved to our order of approximation.*

5. *Instability of Einstein's Universe.*—Setting $p = 0$ in (4) we have

$$3 \frac{d^2 a}{dt^2} = a(\lambda - 4\pi\rho).$$

For equilibrium (Einstein's solution) we must accordingly have $\rho = \lambda/4\pi$. If now there is a slight disturbance so that $\rho < \lambda/4\pi$, $d^2 a/dt^2$ is positive and the universe accordingly expands. The expansion will decrease the density; the deficit thus becomes worse, and $d^2 a/dt^2$ increases. Similarly if there is a slight excess of mass a contraction occurs which continually increases. Evidently Einstein's world is unstable.

The initial small disturbance can happen without supernatural interference. If we start with a uniformly diffused nebula which (by ordinary gravitational instability) gradually condenses into galaxies,

Eddington, A. S. (1930). On the instability of Einstein's spherical world. *Monthly Notices of the Royal Astronomical Society*, 90, 668-678.

1931-G. Lemaître



Mar. 1931. *Homogeneous Universe of Constant Mass.* 483

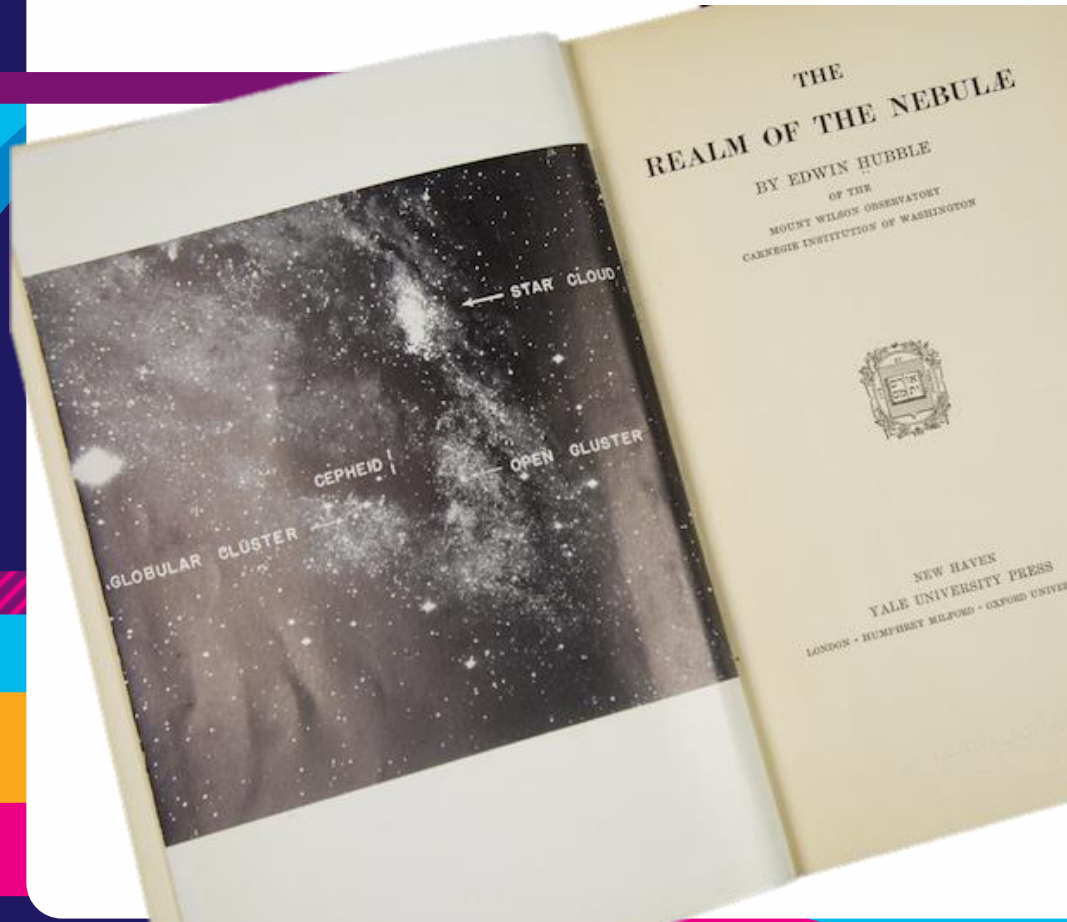
A Homogeneous Universe of Constant Mass and Increasing Radius accounting for the Radial Velocity of Extra-galactic Nebulae. By Abbé G. Lemaître.

(Translated by permission from "Annales de la Société scientifique de Bruxelles,"
Tome XLVII, série A, première partie.)

1. Introduction.

According to the theory of relativity, a homogeneous universe may exist such that all positions in space are completely equivalent; there is no centre of gravity. The radius of space R is constant; space is elliptic *i.e.* of uniform positive curvature $1/R^2$; straight lines are

Hubble vs. Lemaitre



E. Hubble. The realm of the nebulae. 1936

Theories of Cosmology.

CURRENT theories of cosmology employ a model known as the homogeneous, expanding universe of general relativity or, more briefly, as the expanding universe. It is derived from the cosmological equation which expresses a principle of general relativity—that the geometry of space is determined by the contents of space. The equation transcends the body of factual knowledge and can be interpreted and solved only with the aid of assumptions concerning the nature of the universe.

The first solutions, by Einstein and de Sitter (1917), employed the assumptions that the universe is homogeneous and isotropic and also that it is static, i.e., does not vary systematically with time. These solutions were special cases of the general problem and have since been abandoned—Einstein's, because it did not account for red-shifts; de Sitter's, because it neglected the existence of matter. The Einstein universe, it was said, contained matter and no motion, while the de Sitter universe contained motion and no matter. The general problem was first discussed by Friedmann (1922). Subsequently, Robertson (1929) derived the most general formulation (of the line element) from properties of symmetry alone.*

The solution involved the "cosmological constant" and the "radius of curvature of space" as undetermined quantities. By arbitrarily assigning different values to the parameters, various classes of possible universes were described, and among them, it was supposed, the type corresponding to the actual universe would be included. The problem for the observer was to determine

* For further information on this great field of theoretical investigation, the reader is referred to Robertson's authoritative review of the development of the subject up to the end of 1932; "Relativistic Cosmology," *Reviews of Modern Physics*, 5, 1, 1933. A complete bibliography of the more important contributions, with short descriptions of their contents, is included, and also a list of recommended, nontechnical discussions of the field. Among the latter is an exceptionally clear statement from the mathematical point of view, by Robertson himself ("The Expanding Universe," *Science*, 76, 221, 1932).

Hubble vs. Lemaitre

COMMENT

ART Scientific chess verify a rediscovered painting by Leonardo da Vinci **116**

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SCIENCE Immunologists question priority for this year's prize in medicine **118**



Georges Lemaitre giving a lecture at the Catholic University of Louvain in Belgium.

Mystery of the missing text solved

A discovered letter explains the loss of key paragraphs during the translation of one of Georges Lemaitre's papers about the expanding Universe, shows **Mario Livio**.

A passionate debate has flared up in recent months about who deserves the credit for one of the most profound discoveries of our time: that our Universe is expanding, and so had a "beginning". The American astronomer Edwin Hubble, who tracked the expansion in the velocities and distances of scores of distant galaxies during the 1920s, is usually cited. But a few articles have raised the suspicion that someone concealed a key paper by the Belgian priest and cosmologist Georges Lemaitre to ensure Hubble's priority.

There is little doubt that Lemaitre deserves the credit for proposing an expanding Universe. But the censorship charges tarnish Hubble's genuine achievement of confirming and extending the idea. As someone intimately involved with Hubble's nameplate — the Hubble Space Telescope — I became intrigued by this "whodunnit" mystery, and decided to investigate. As a result, I unearthed

a letter from Lemaitre that, to my satisfaction, ends the debate. Here are the background facts. By February 1922, American astronomer Vesto Slipher had measured the redshifts (frequency shifts indicating relative motions) for 41 galaxies (then known as nebulae) in the northern sky. Listing them in his 1923 book *The Mathematical Theory of Heterogeneity*, British physicist Arthur Eddington noted that: "The great preponderance of positive [receding] velocities is very striking." But he added that a lack of observations of southern nebulae prevented him from drawing further conclusions.

In 1927, Lemaitre published, in French, a remarkable paper in the relatively obscure *Annales de la Société Scientifique de Bruxelles*. It was entitled (in its English translation): "A homogeneous Universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulae". In it, Lemaitre reported his discovery of dynamic solutions to Einstein's general relativity equations, from which he derived what

Louvain, le 9 mars 1931

SMART

precipitate the honour for me and for our society paper reprinted by the Royal Astronomical Society a translation of the paper. I did not find it is clearly of no actual interest, and also a note, which could be replaced by a small bilingual note and new papers on the subject. I join with indication of the passages omitted in it. I made this translation as exact as I can, but I am glad if some of yours would be kind enough to correct my English which I am afraid is rather poor. It is changed, and even the final suggestion is not written again the table which may be printed.

In addition on the subject, I just obtained the influence of the condensations and the possible expansion. I would be very glad to have to your society as a separate paper.

It is very much to become a fellow of your society to be presented by Prof. Eddington and

Eddington has yet a reprint of his May paper in a very glad to receive it.

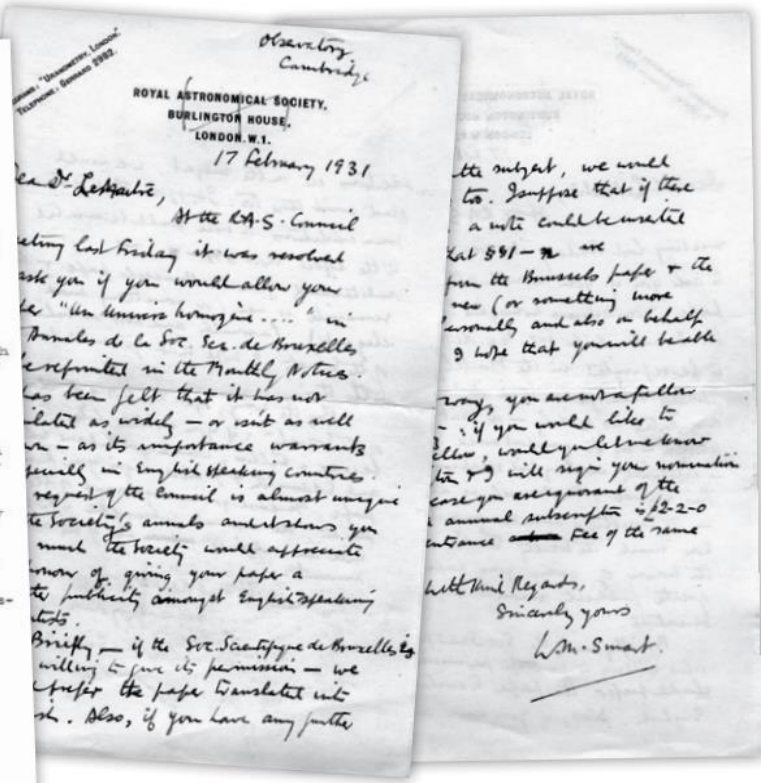
Kind enough to present my best regards to profes-

sor

yours sincerely

G. Lemaitre

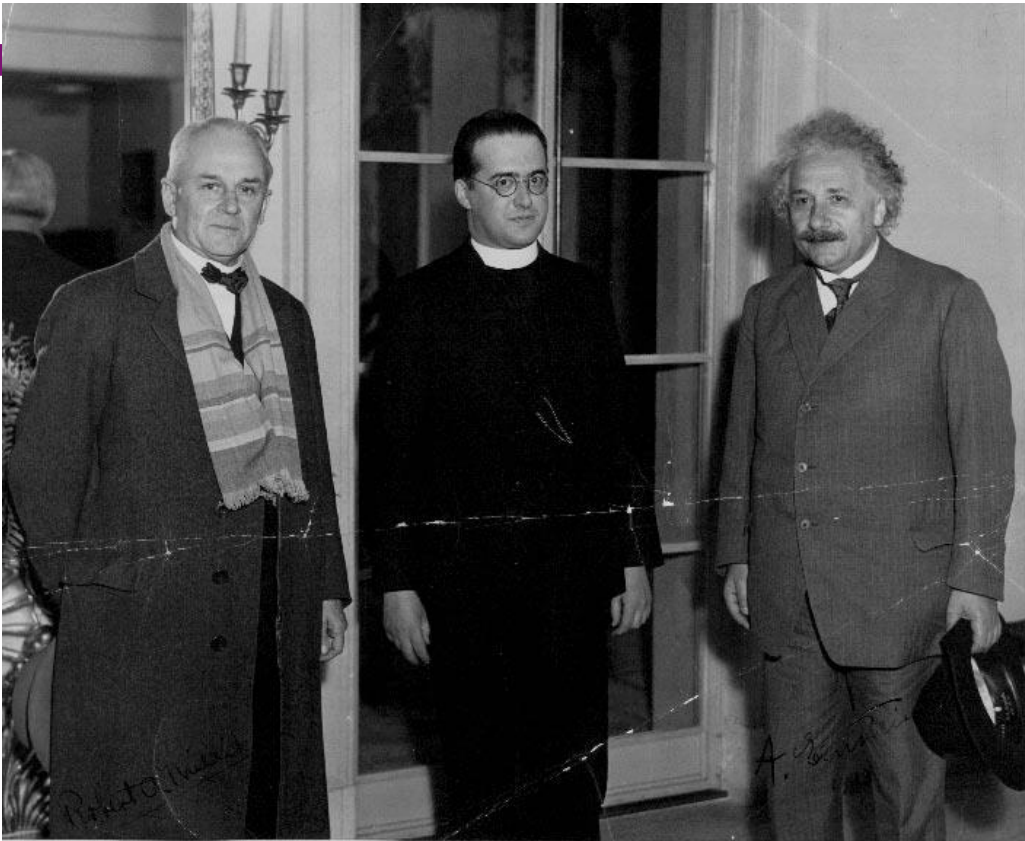
40 rue de Namur
Louvain



Letters between Georges Lemaitre and William Smart reveal that there was no conspiracy behind the removal of paragraphs from a translated paper.

Livio, M. (2011). Lost in translation: Mystery of the missing text solved. *Nature*, 479(7372), 171-173.

1933-Pasadena



“Esta es la explicación más hermosa y satisfactoria de la creación que haya escuchado jamás”

1933-Pasadena

THE NEW YORK TIMES MAGAZINE, FEBRUARY 19, 1933.

3

LEMAITRE FOLLOWS TWO PATHS TO TRUTH

By DUNCAN AIKMAN

PASADENA.

"THERE is no conflict between religion and science," the Abbé Lemaitre has been telling audiences over and over again in this country and then proving it by explaining the aims of both. His view is interesting and important not because he is a Catholic priest, not because he is one of the leading mathematical physicists of our time, but because he is both. Here is a man who believes firmly in the Bible as a revelation from on high, but who develops a theory of the universe without the slightest regard for the teachings of revealed religion on genesis. And there is no conflict!

Such an attitude would have been preposterous to a Victorian physicist. Either you accept the whole Book of Genesis and therefore shut yourself out of the world of science, or you accept science and repudiate the prophets as expositors of the manner in which the universe began. Today the physicist is meeker. Behind his formulas there is something that is still veiled. He is half mystic and ready to admit that the universe may reveal itself in other ways than in mathematical equations or the hands and lines of a spectograph. The abbé, therefore, follows the trend of modern thinking and derives from it more than ordinary satisfaction because he happens to be trained in theology as well as in mathematical physics.

Lemaitre, like Eddington, finds that science and religion supplement each other. Science can never study the universe as a whole. It selects a small portion, as much as it can handle, and then makes deductions. To a cosmologist the earth and Mars are only planets wheeling around the sun. Are they inhabited? Are they washed by air and water? Why were they created? Is there purpose in the universe? Science is indifferent to such questions, but not

The Famous Physicist, Who Is Also a Priest, Tells Why He Finds No Conflict Between Science and Religion

that there must be authentic religious dogma in the binomial theorem. Nevertheless a lot of otherwise intelligent and well-educated men do go on believing or at least acting on such a belief. When they find the Bible's scientific references wrong, as they often are, they repudiate it utterly. Should a priest reject relativity because it

that it took perhaps ten thousand million years to create what we think is the universe. Genesis is simply trying to teach us that one day in seven should be devoted to rest, worship and reverence—all necessary to salvation."

"And that story about Jonah and the big fish?"

"I admit that a whale cannot

powers with which they are credited in the Bible.

"If scientific knowledge were necessary to salvation," he says, "it would have been revealed to the writers of the Scriptures and they would have set it down in their verses. For instance, the doctrine of the Trinity is much more abstruse than anything in relativity or

question of salvation. On other questions they were as wise or as ignorant as their generation. Hence it is utterly unimportant that errors of historic and scientific fact should be found in the Bible, especially if errors relate to events that were not directly observed by those who wrote about them. The idea that because they were right in their doctrine of immortality and salvation they must also be right on all other subjects is simply the fallacy of people who have an incomplete understanding of why the Bible was given to us at all."

Lemaitre tells of a classroom scene in which he figured. An old father was expounding at the desk. Before him sat the lad who was to discover the expanding universe and who, even then, was brimful of science. In his eagerness the lad read into a passage of Genesis an anticipation of modern science.

"I pointed it out," says Lemaitre,

"but the old Father was skeptical.

"If there is a coincidence," he decided, "it is of no importance. Also if you should prove to me that it exists I would consider it unfortunate. It will merely encourage more thoughtless people to imagine that the Bible teaches infallible science, whereas the most we can say is that occasionally one of the prophets made a correct scientific guess."

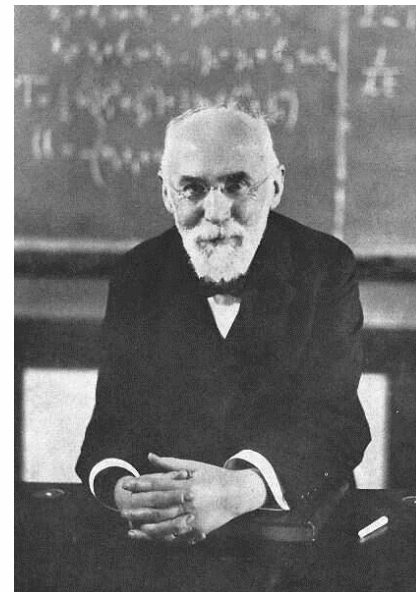
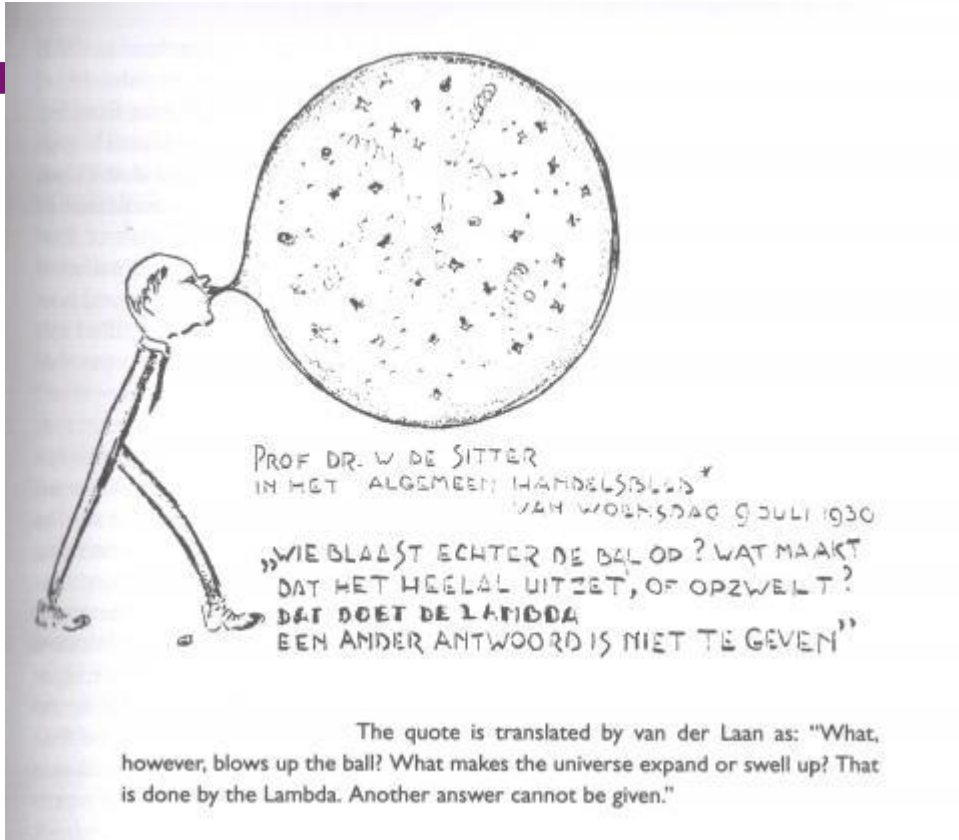
THERE is, the abbé admits, a varying sense of conflict between science and religion in the different branches of science.

"The biologists seem to have peculiar difficulties," he reasons. "There is every reason for this. They have only recently discovered a few guiding laws and principles. Hence, in the past their studies have been confusing rather than enlightening. In a way their subject-matter has been gross.

"But give the biologist more laws like those of the Abbé Mendel and a new spirit is bound to awaken. The sense that this is a merely



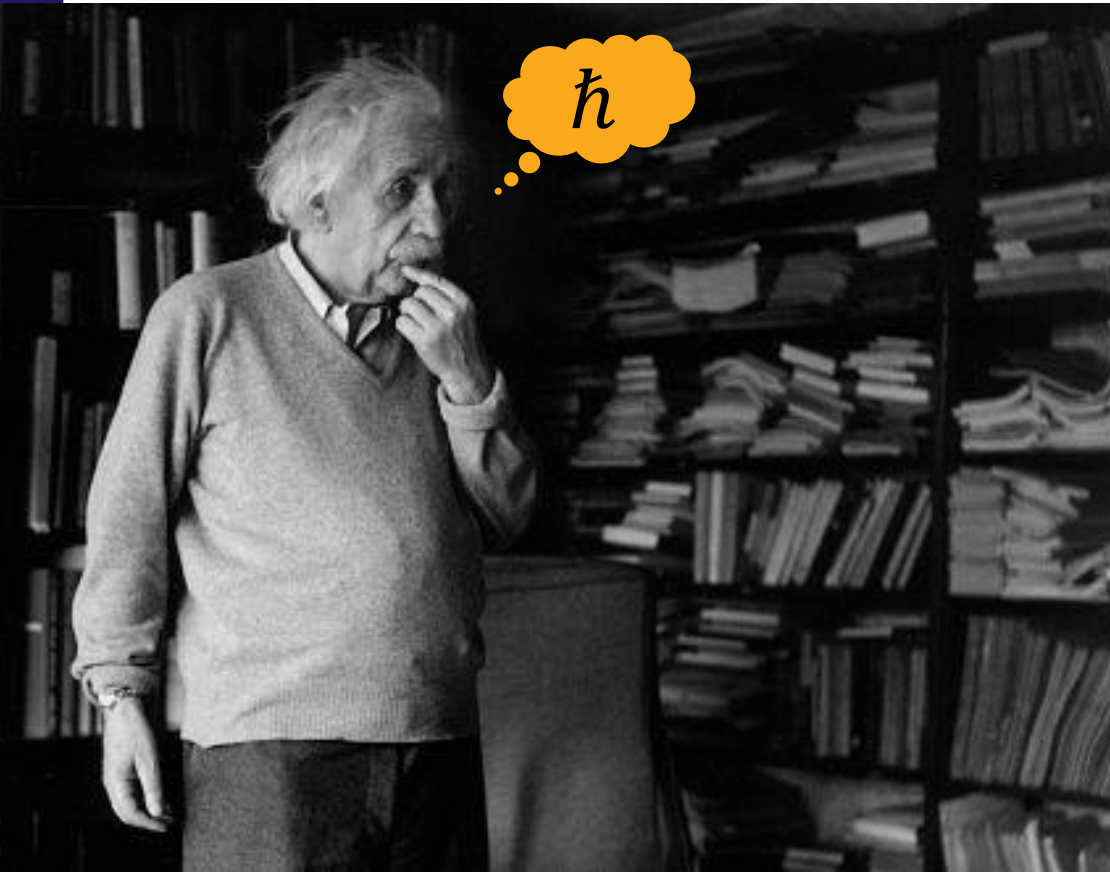
de-Sitter



Algo tan feo

“Desde que introduje este término he tenido mala conciencia. Pero en aquel momento no pude ver otra manera de contemplar el hecho de la existencia de una densidad media finita de materia. Me pareció verdaderamente feo que la ley del campo de gravitación estuviera compuesta de dos términos lógicamente independientes y enlazados por una suma. Sobre la justificación de estas impresiones acerca de la simplicidad lógica es difícil argumentar. No puedo evitar sentir las con fuerza y soy incapaz de creer que se dé en la naturaleza algo tan feo”

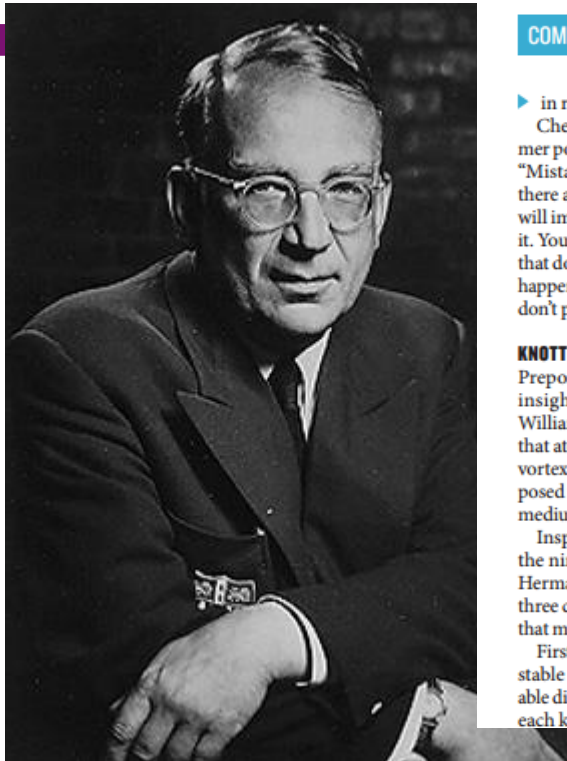
Mi mayor error



“Einstein me comentó
hace muchos años que
la idea de la repulsión
cósmica había sido el
mayor error de su vida”

G. Gamow

Mi mayor error



COMMENT

► in recognizing the necessity of blunders. Chemist Linus Pauling knew it. His former postdoc, Jack Dunitz, recalls being told: "Mistakes do no harm in science because there are lots of smart people out there who will immediately spot a mistake and correct it. You can only make a fool of yourself and that does no harm, except to your pride. If it happens to be a good idea, however, and you don't publish it, science may suffer a loss."

KNOTTY PROBLEM

Preposterous ideas can lead to important insights. In 1867, the eminent physicist William Thomson (Lord Kelvin) proposed³ that atoms were not point-like but 'knotted vortex tubes of the ether'. Ether was the supposed fluid that pervaded space, providing a medium for electricity and magnetism.

Inspired by work on vortices in fluids by the nineteenth-century German physicist Hermann von Helmholtz, Kelvin identified three characteristics of knotted vortex tubes that made them attractive models for atoms.

First, vortices in fluids were astonishingly stable — mirroring to Kelvin the "unalterable distinguishing qualities" of atoms — and each knot could be classed according to its

HISTORY REVISITED

Did Einstein ever say "biggest blunder"?

Almost any history of Albert Einstein's 'cosmological constant' mentions his "biggest blunder"—the introduction of this constant to counteract gravity into equations characterizing the Universe.

Did Einstein actually say this? After scrutinizing dozens of documents while researching my book *Brilliant Blunders* (Simon & Schuster, 2013), I found no evidence that he did.

The "biggest blunder" phrase seems to have come from the colourful physicist

George Gamow in an article published in the September 1956 issue of *Scientific American*. Gamow later repeated the story in his 1970 autobiography, *My World Line*.

Einstein was indeed unhappy about having introduced the cosmological constant, saying in a letter to cosmologist Georges Lemaître that he was "unable to believe that such an ugly thing should be realized in nature". Calling it the "biggest blunder" was, in my view, Gamow's hyperbole.

and recombine the four ends in various ways, which can be described using knot theory.

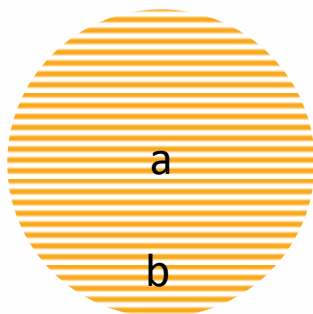
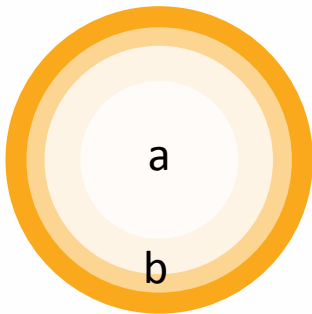
EXTRAORDINARY CLAIMS

Blunders are sometimes hard to correct. Modern experiments can be so intricate and require such big investments in time and funds that replicating them becomes prohibitive. When a result is widely assumed

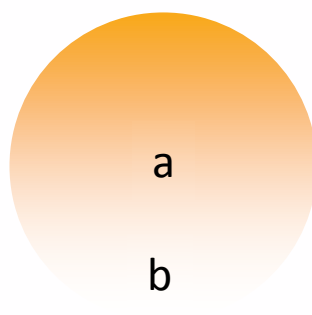
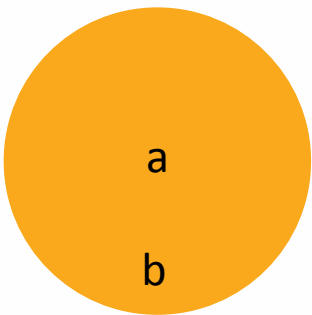
impact-driven atmosphere? I believe it must. We should make space for risky scientific proposals in grant and evaluation processes.

Until a decade ago, the committees that allocated observing time on the Hubble Space Telescope were encouraged to give up to 10% of the time to proposals with a low probability of success but potentially high return. A similar philosophy could be

Universo homogéneo e isótropo

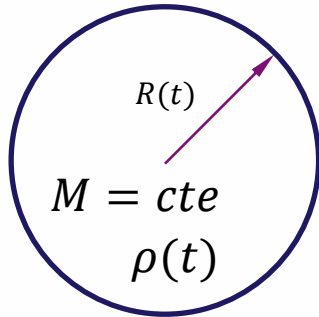


Homogéneo: todos los puntos del espacio tienen las mismas propiedades físicas.



Isótropo: todas las direcciones del espacio con respecto a un punto cualquiera muestran las mismas propiedades físicas y geométricas en cada instante.

Aproximación clásica



$$F = -\frac{GMm}{R^2}$$

$$\ddot{R} = -\frac{GM}{R^2}$$

$$\int \ddot{R}\dot{R}dt = -\int \frac{GM}{R^2}\dot{R}dt$$

$$\frac{\dot{R}^2}{2} = \frac{GM}{R} + U \quad U = cte$$

$$M = cte = \frac{4\pi R^3 \rho(t)}{3}$$

para una expansión isotrópica

$$R(t) = a(t)r \quad r = \text{radio comóvil}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho(t)}{3} + \frac{2U}{r^2 a^2}$$

1ra. ec. de Friedmann

Aproximación clásica

tres soluciones:

$U > 0 \rightarrow \frac{da}{dt} > 0$ siempre distinto de cero, el universo siempre se expandirá

$U < 0 \rightarrow \frac{da}{dt} = 0$ $a_{max} = \sqrt{-Ur/GM}$ la expansión se detendrá y se convertirá en contracción

$$U = 0 \rightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho(t)}{3} \rightarrow \rho(t) = \frac{3M}{4\pi a^3 r^3}$$

Aproximación clásica

$$dQ = dE + pdV \quad \text{1ra. Ley Termodinámica}$$

$$dQ = 0 \quad \dot{E} = V \left(\dot{\varepsilon} + 3 \frac{\dot{a}}{a} \varepsilon \right)$$

$$\dot{E} + p\dot{V} = 0$$

$$V(t) = \frac{4\pi}{3} r^3 a(t)^3 \quad \dot{\varepsilon} = +3 \frac{\dot{a}}{a} \varepsilon + 3 \frac{\dot{a}}{a} p = 0$$

$$\dot{V} = 3V \left(\frac{\dot{a}}{a} \right)$$

$$E(t) = V(t)\varepsilon(t)$$

utilizando 1ra. ec. de Friedmann y ec. de Einstein

$$2\dot{a}\ddot{a} = \frac{8\pi G}{3c^2} (\dot{\varepsilon}a^2 + 2\varepsilon a\dot{a})$$

$$\varepsilon(t) = \rho(t)c^2$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\varepsilon + 3p)$$

2da. ec. de Friedmann

Cte. cosmológica

$$\nabla^2 \Phi + \Lambda = 4\pi G \rho$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho(t)}{3} + \frac{2U}{r^2 a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\varepsilon + 3p) + \frac{\Lambda}{3}$$

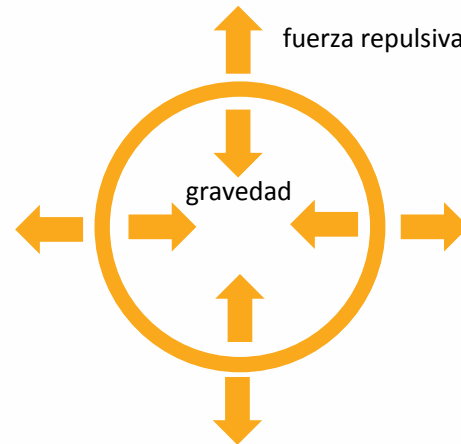
$$\varepsilon_\Lambda = \frac{c^2}{8\pi G} \Lambda \quad p_\Lambda = -\frac{c^2}{8\pi G} \Lambda$$

si $\varepsilon > 0$ aceleración negativa.

$p > 0$ aporta aceleración negativa.

(universo homogéneo, no hay gradientes de presión)

$p < 0$ produce aceleración.



Ecuación de estado

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\varepsilon(t)}{3c^2} + \frac{2U}{r^2 a^2} \quad \text{1ra. ecuación de Friedmann}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\varepsilon + 3p) \quad \text{2da. ecuación de Friedmann}$$

$a(t), \varepsilon(t), p(t)$ 2 ecuaciones 3 incógnitas

ecuación de estado

$$p = \omega\varepsilon$$