
CAOS

¿QUÉ ES EL CAOS?

Características de los sistemas caóticos

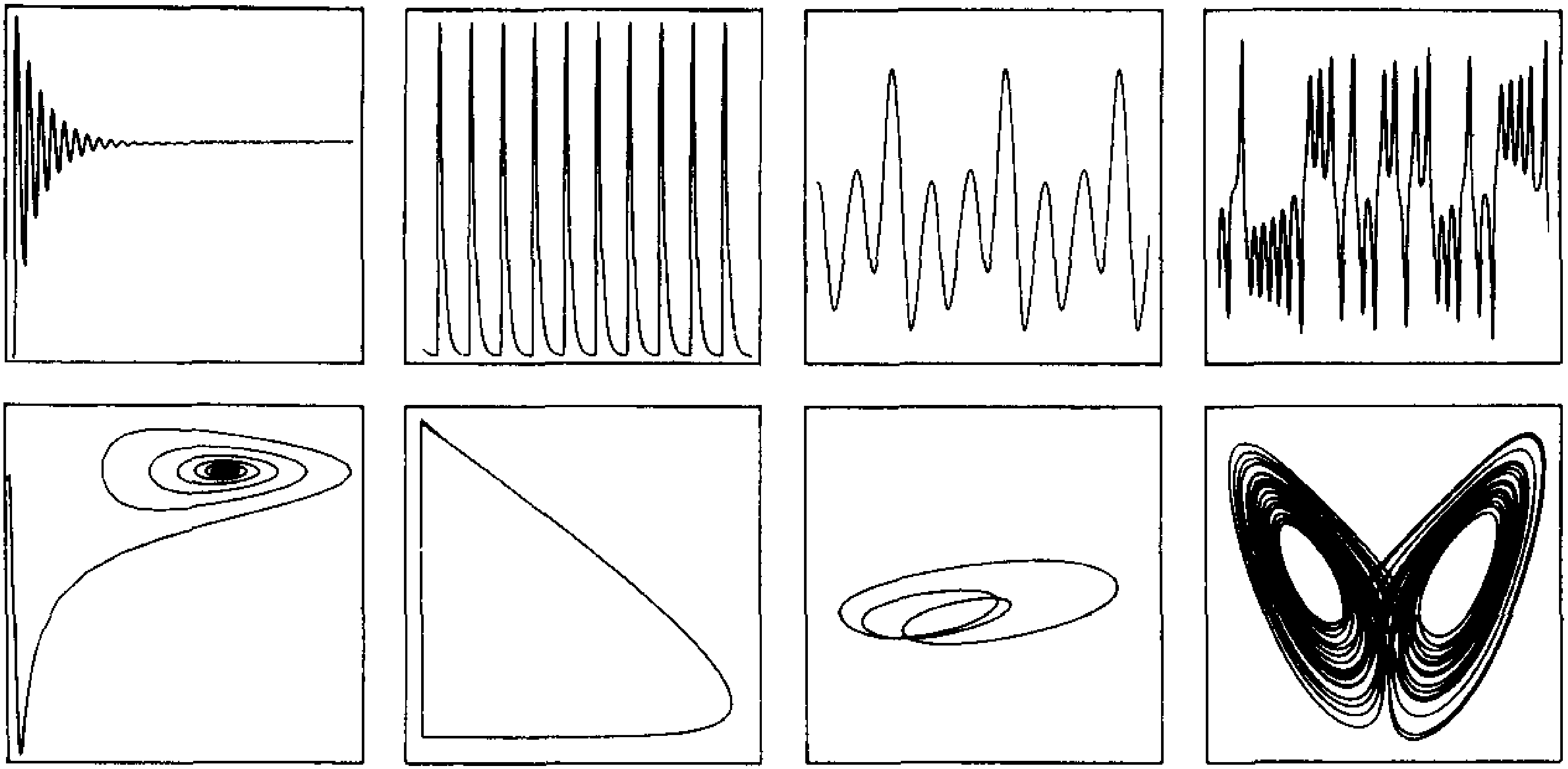
- Sistemas dinámicos no lineales
- Sensibilidad alta a las condiciones iniciales
- Incapacidad para predecir resultados a largo plazo
- Atractores
- Presencia de bifurcaciones

SISTEMAS ESTABLES VS ALEATORIOS VS CAÓTICOS

ENTRADA	SALIDA DE UN SISTEMA ESTABLE	SALIDA DE UN SISTEMA ALEATORIO	SALIDA DE UN SISTEMA CAÓTICO
5	22.5	4 (??)	10
5.001	22.503	25	37
10	45	9	29
5	22.5	46 (??)	10

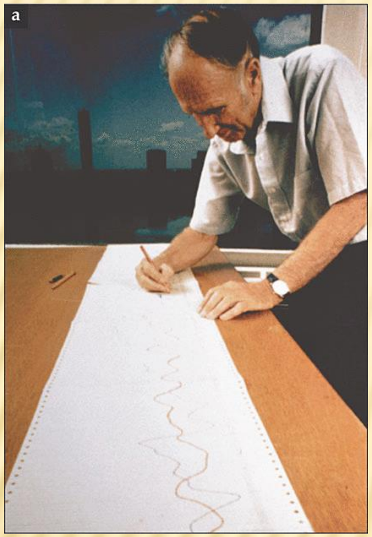
- Así es el sistema caótico: Determinista, Ultrasensible e Impredecible a largo plazo

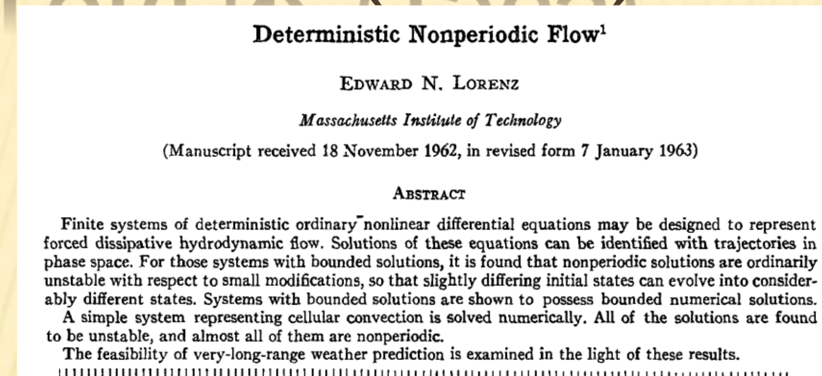
ESPACIO DE FASE



Irving R. Epstein

EDWARD LORENZ (1963)





Deterministic Nonperiodic Flow[1]

Edward N. Lorenz

Massachusetts Institute of Technology

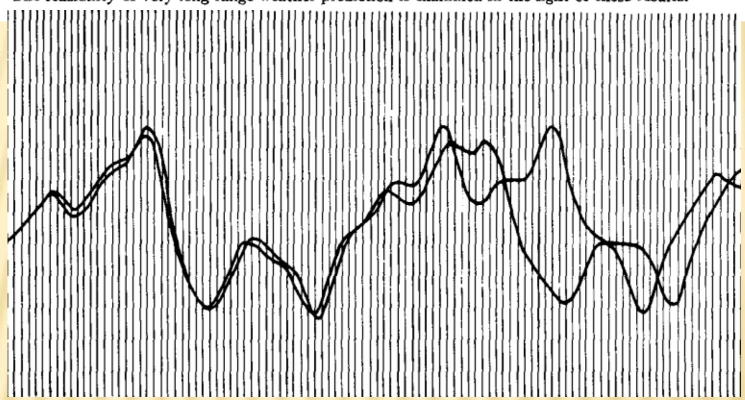
(Manuscript received 18 November 1962, in revised form 7 January 1963)

Abstract

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.



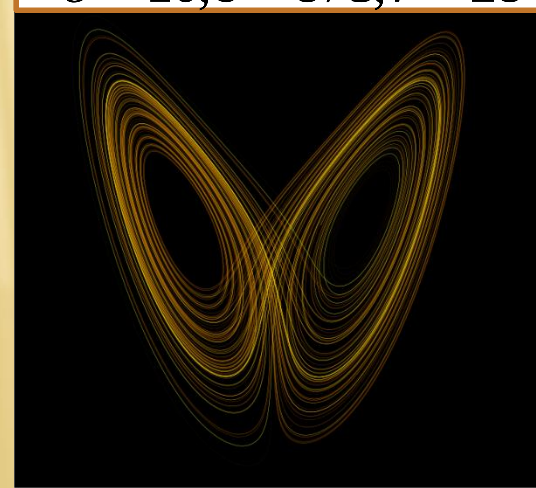
Ecuaciones de Convección

$$\frac{dX}{dt} = -\sigma X + \sigma Y$$

$$\frac{dY}{dt} = -XZ + rX - Y$$

$$\frac{dZ}{dt} = XY - bZ$$

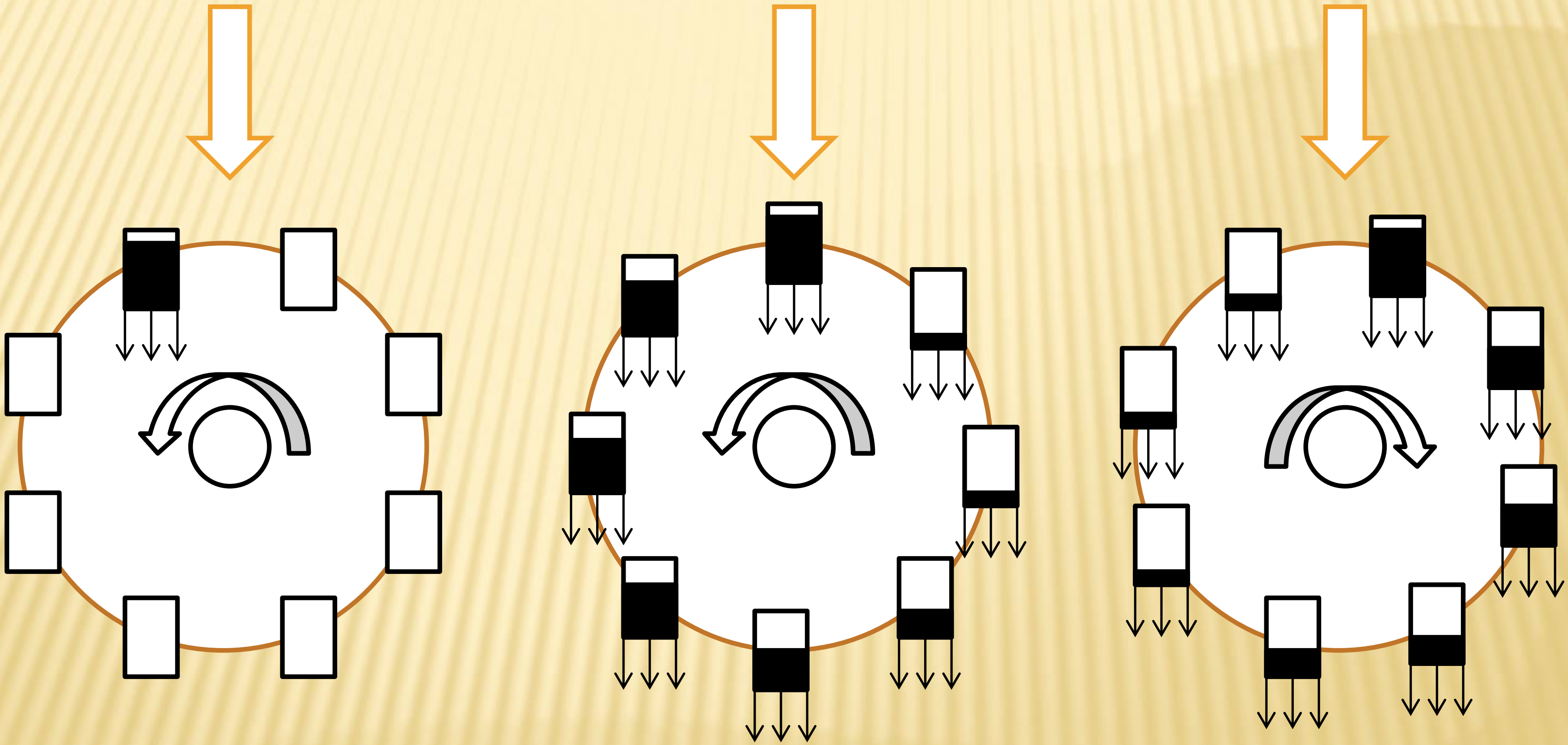
$$\sigma = 10, b = 8/3, r = 28$$



- No periodicidad ⟷ Dependencia sensible
- Efecto Mariposa

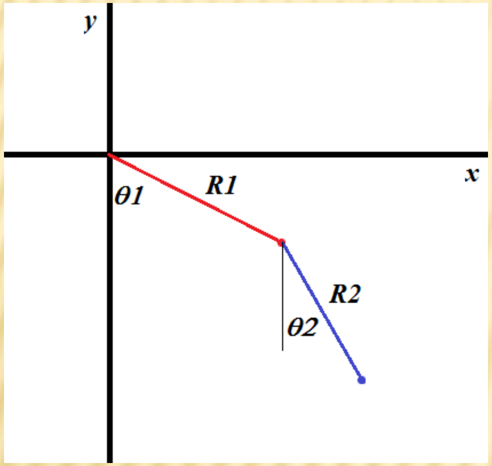
1972 - “Does the flap of a butterfly’s wings in Brazil set off a tornado in Texas?”

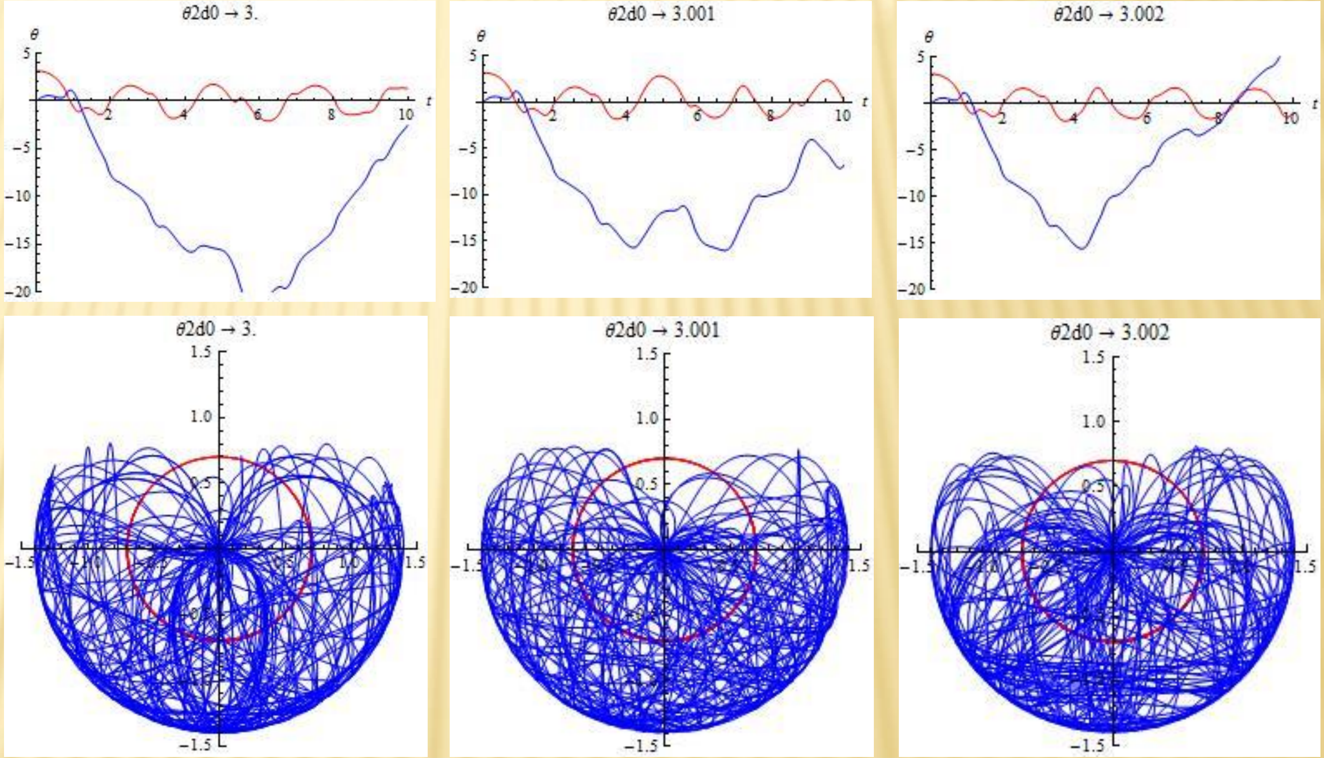
THE LORENZIAN WATERWHEEL



PÉNDULO DOBLE

Sensibilidad a las condiciones iniciales



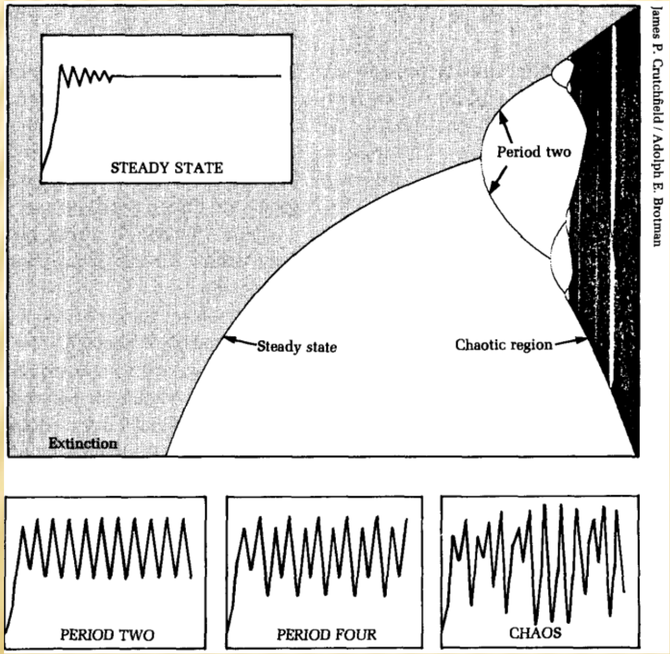


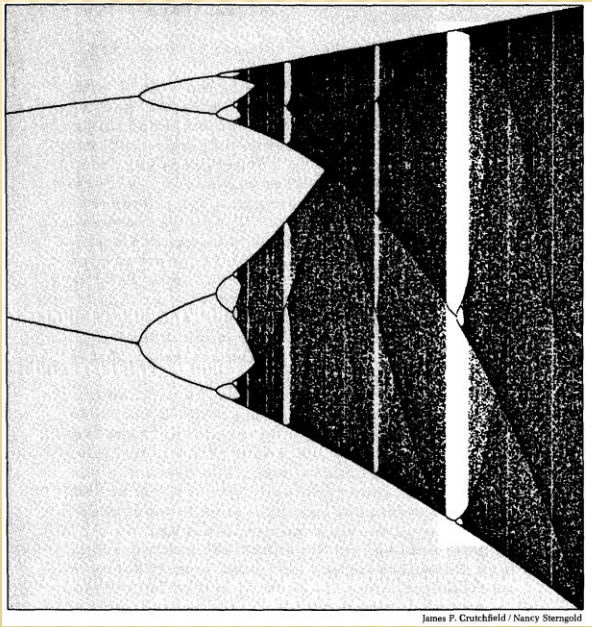
BIFURCACIONES

- Robert May, biólogo
- Poblaciones individuales a lo largo de los años

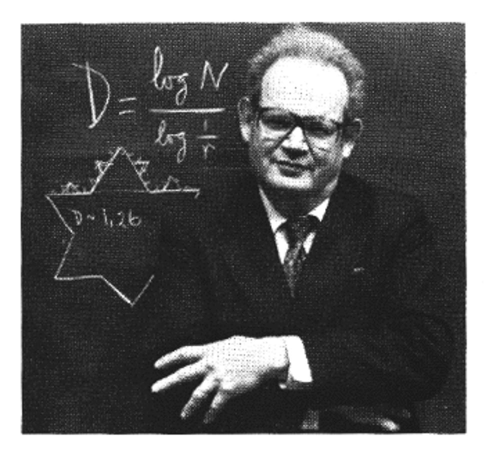
$$x_{n+1} = r x_n (1 - x_n)$$

$$\lim_{n \to \infty} \frac{x_{n+1} - x_n}{x_{n+2} - x_{n+1}} = 4{,}669$$

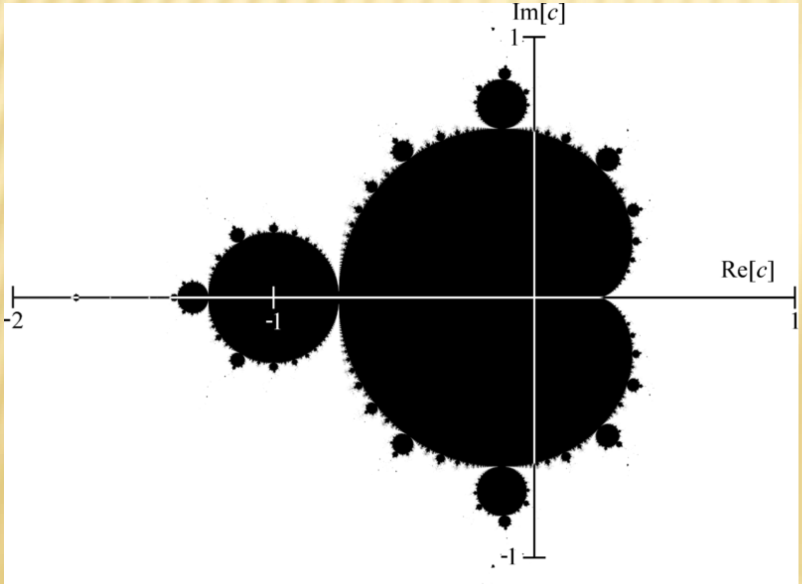




FRACTALES



- Benoît Mandelbrot (1924-2010)
- ¿Qué tan larga es la costa de Gran Bretaña?
- ¿Cuál es la dimensión de un ovillo de lana?

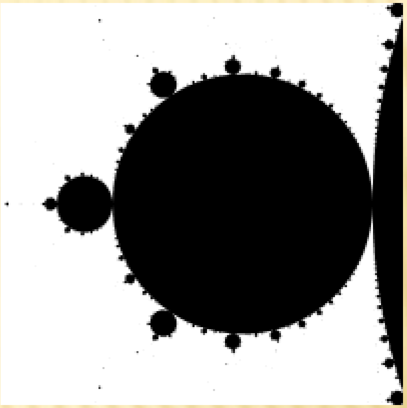


- Conjunto de Mandelbrot

$$z_0 = 0$$
$$z_{n+1} = z_n^2 + c$$

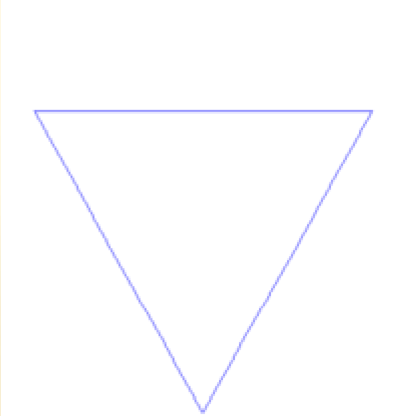
AUTOSIMILARIDAD

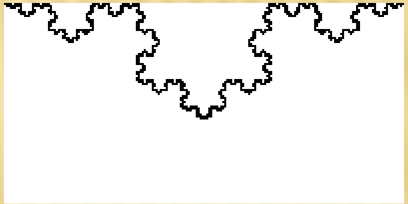
- Conjunto de Mandelbrot



- Copo de nieve de Koch

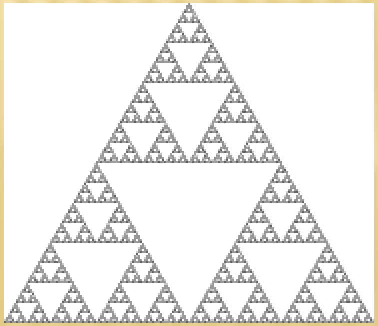
  d=1,28





- Triángulo de Sierpinski

  d=1,58



GRACIAS